

Agenda

How a computer runs a program

Von Neuman architecture overview

Building a CPU

- Logic gates

- Circuits

- ALU (Addition, Subtraction, Multiplexor)

- Storage (R-S Latches, registers)

- Control

Recall: Building and Running a C program

1. **Compiling** a C program translates it to binary (0's and 1's)

- The binary file is an **executable**, meaning “we can run it”

C program:

```
// example C program
int main() {
    int x = 6 + 7;
    printf("x %d", x);
    return 0;
}
```

gcc
compiler

binary executable program:

```
01010110101
01010101010
10101010101
01010100
```

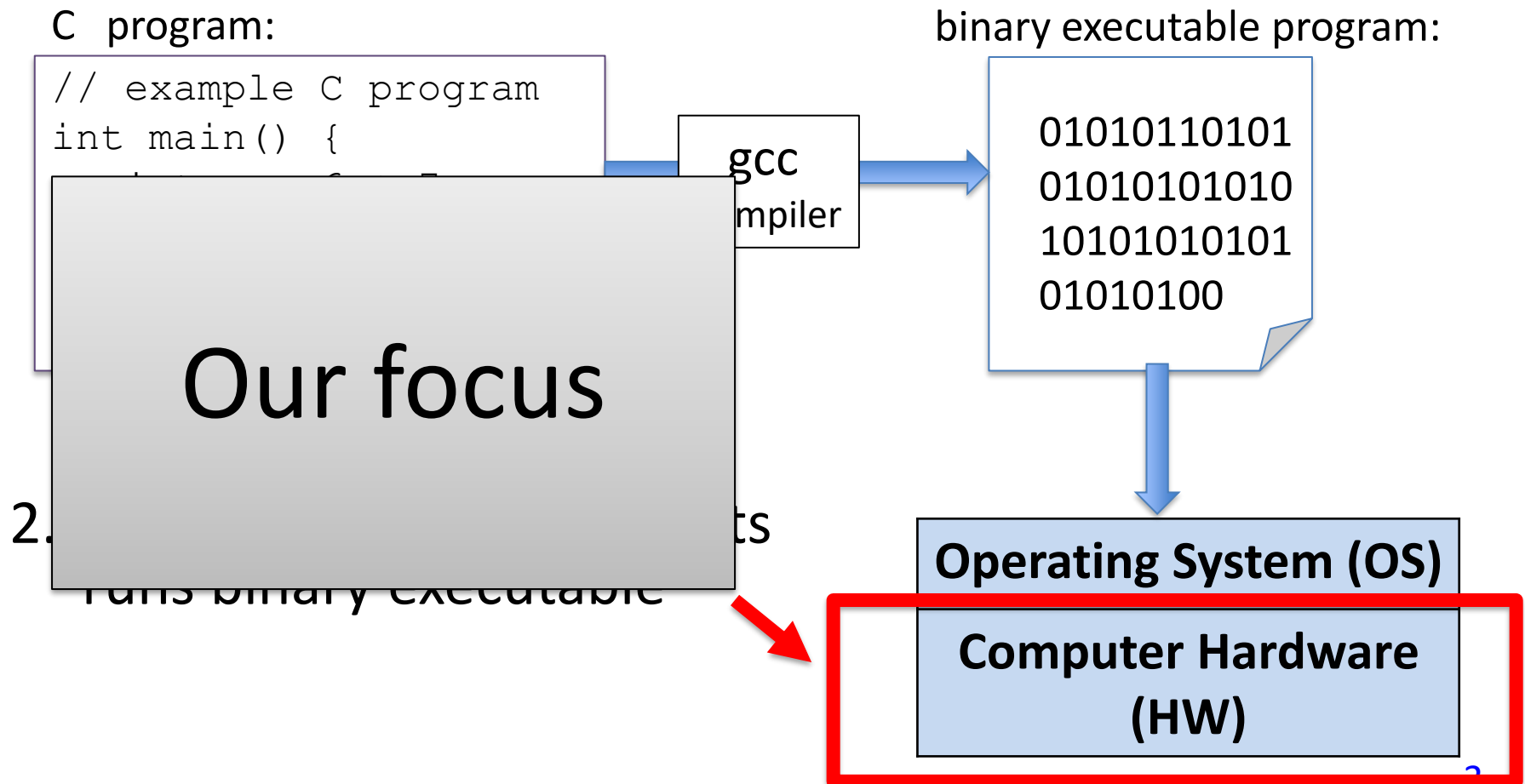
2. With OS's help, HW circuits runs binary executable

Operating System (OS)
Computer Hardware (HW)

Recall: Building and Running a C program

1. **Compiling** a C program translates it to binary (0's and 1's)

- The binary file is an **executable**, meaning “we can run it”



Von Neumann Architecture (1945)

Computer is a generic computing machine

- Can be used to compute anything that is computable
- Based on Alan Turing's Universal Turing Machine

Uses a **stored program model**

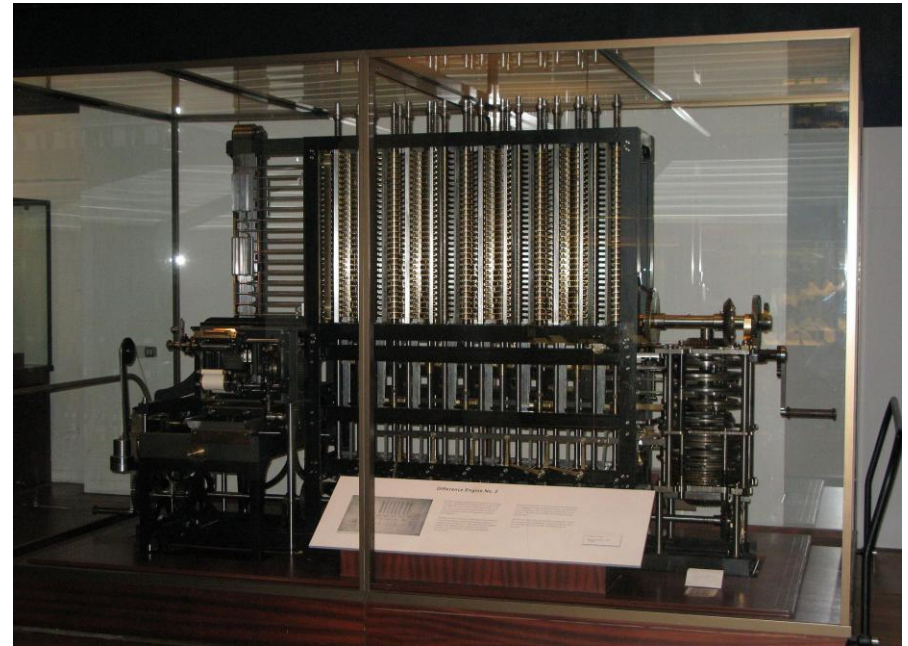
- Both program & data loaded into computer memory
- No distinction between data & instructions in memory

*All modern computers based
on the Von Neumann model*

Early computers

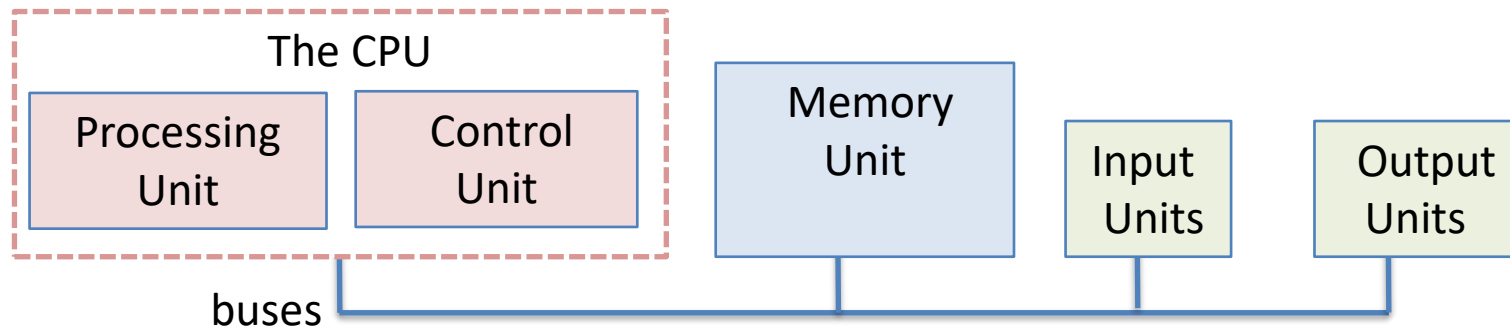
Earlier computers used fixed program encoded on machine, data loaded and run by fixed program

Left, The difference engine (Babbage) could estimate polynomial functions useful for making tables



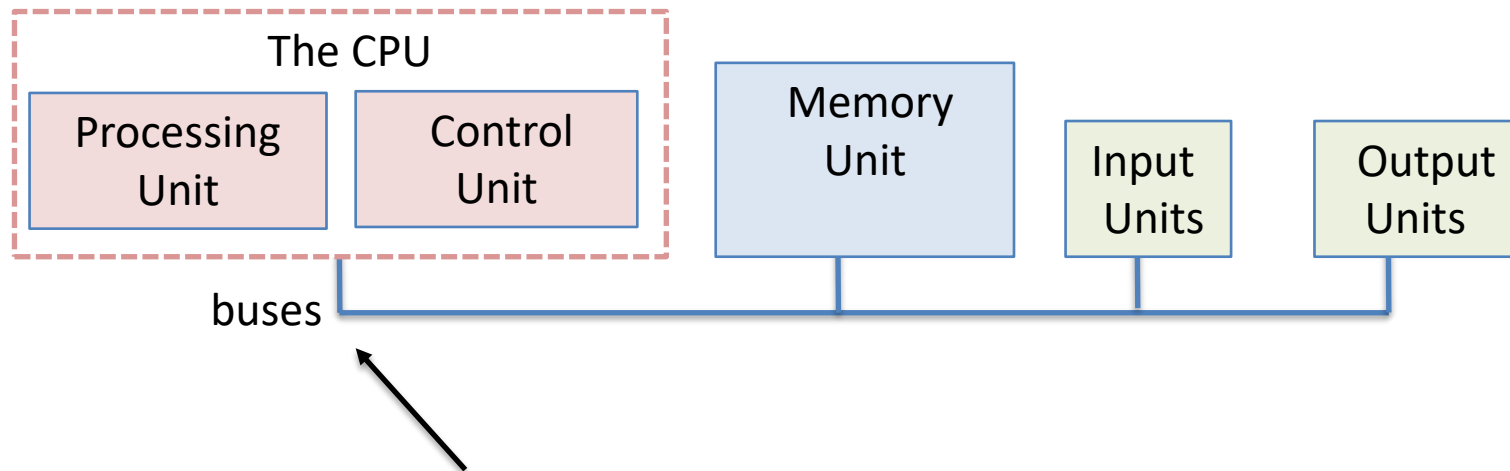
Von Neumann Model

units connected by buses (wires) to communicate



Von Neumann Model

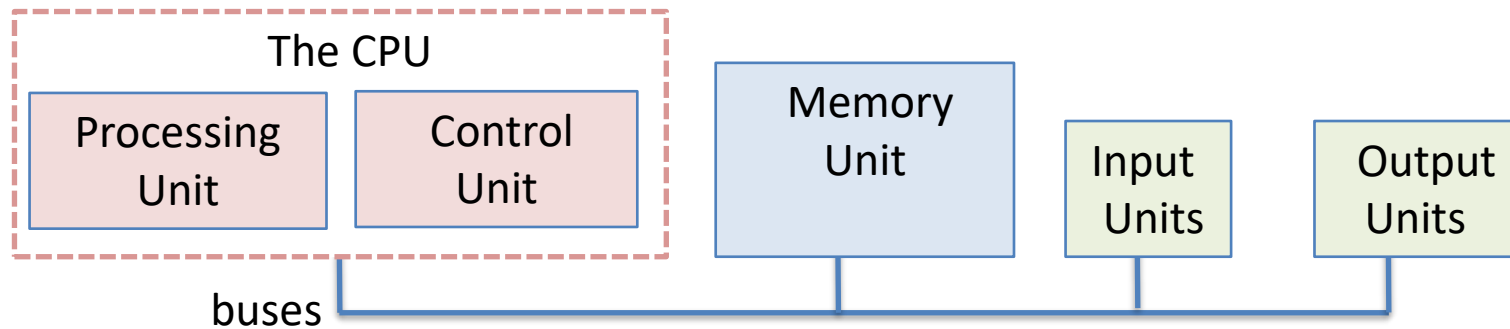
5 units connected by buses (wires) to communicate



Processing & Control Units: implement CPU
execute program instructions on program data

Von Neumann Model

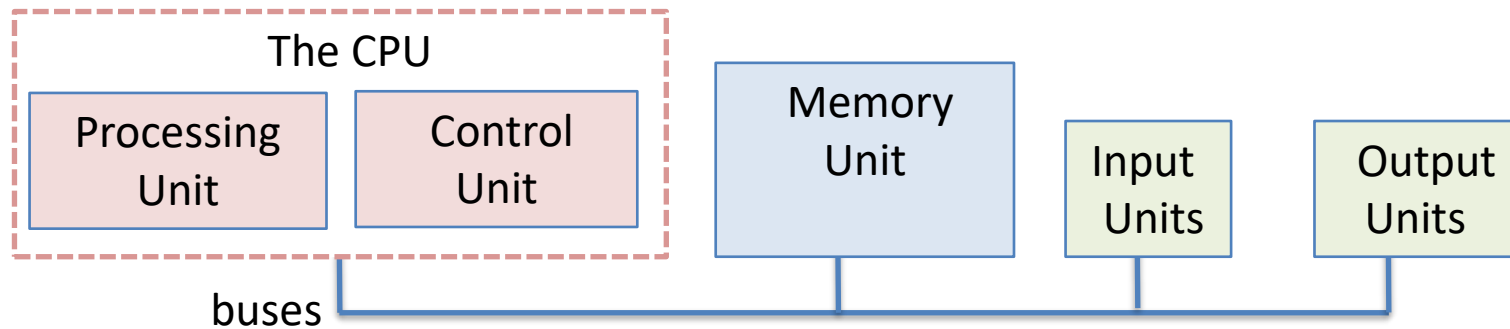
5 units connected by buses (wires) to communicate



Memory: stores program instructions and data
memory is addressable: addr 0, 1, 2, ...

Von Neumann Model

5 units connected by buses (wires) to communicate

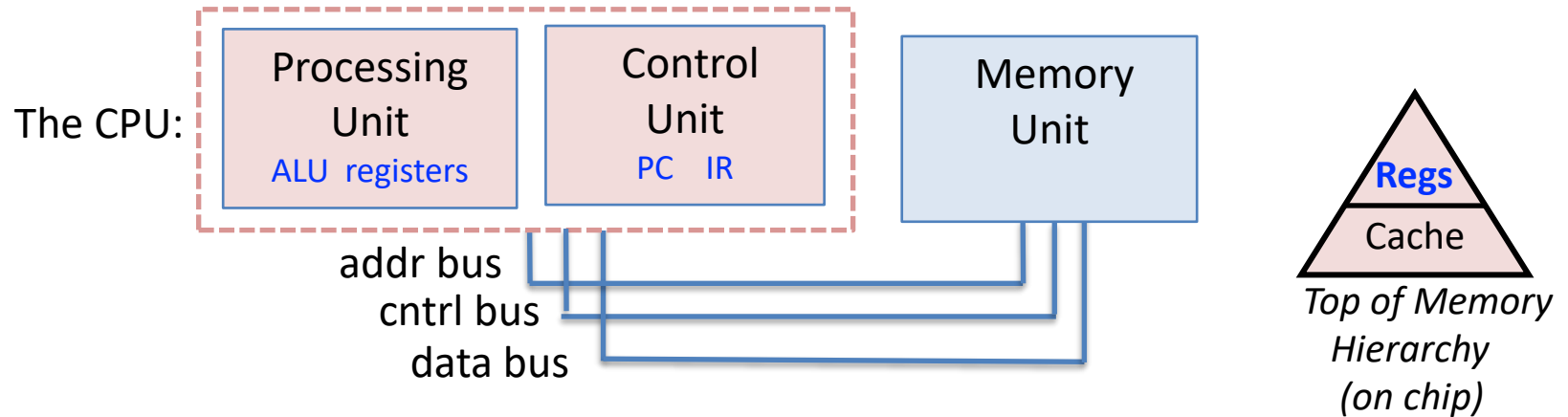


Input, Output: interface to computer

trigger actions: load program, initiate execution, ...

display/store results: to terminal, save to disk, ...

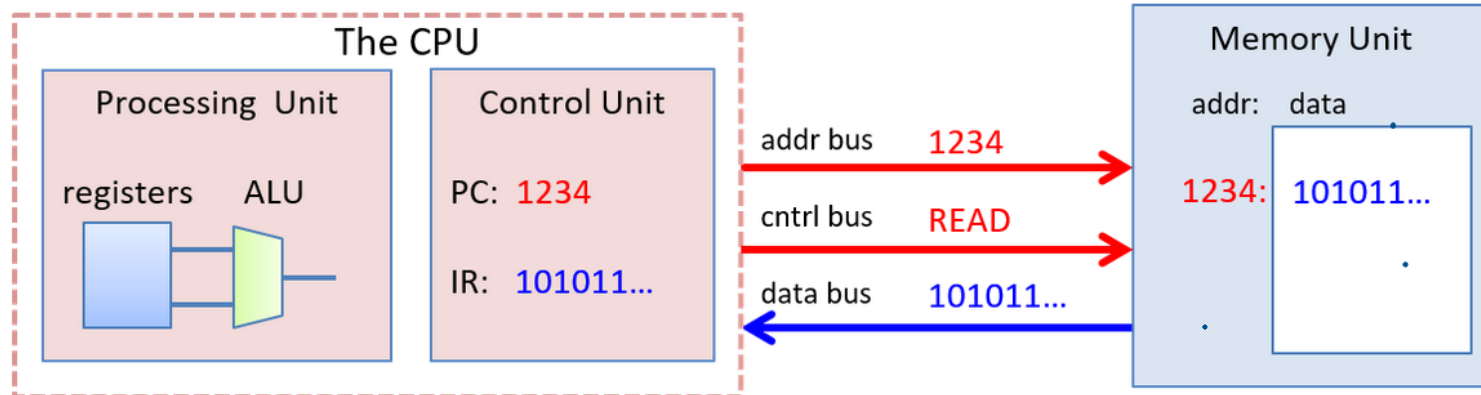
Von Neumann Model: CPU/Memory



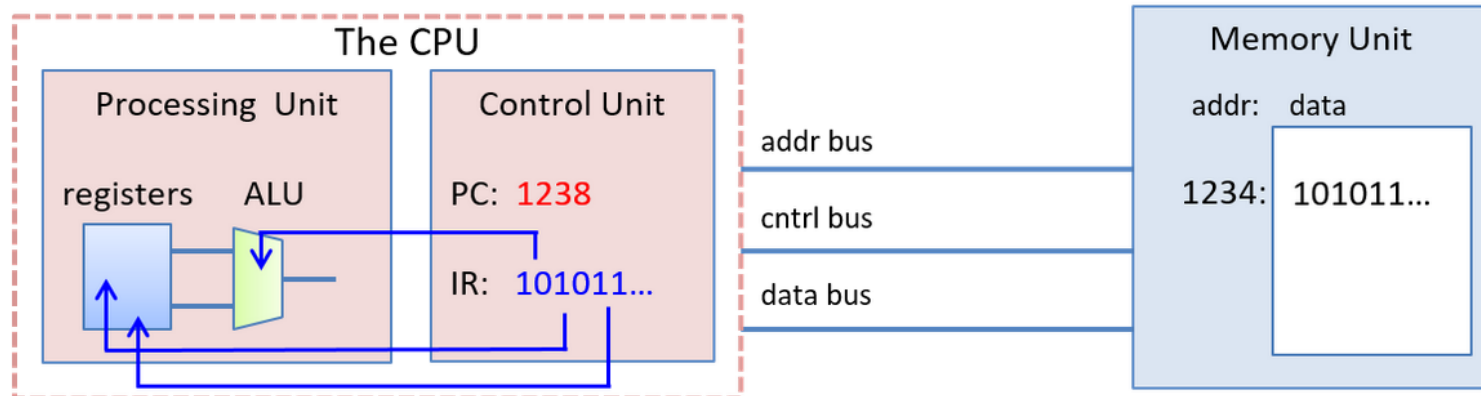
Instruction Execution: controlled by Control Unit

1. **Fetch** instruction from Memory (its addr in **PC**) into **IR** (and increment address in PC to next instruction address)
2. **Decode** instruction bits to determine operation & operands
3. **Execute** instruction on **ALU**
4. **Store** instruction results to Memory

Example: Adding two numbers

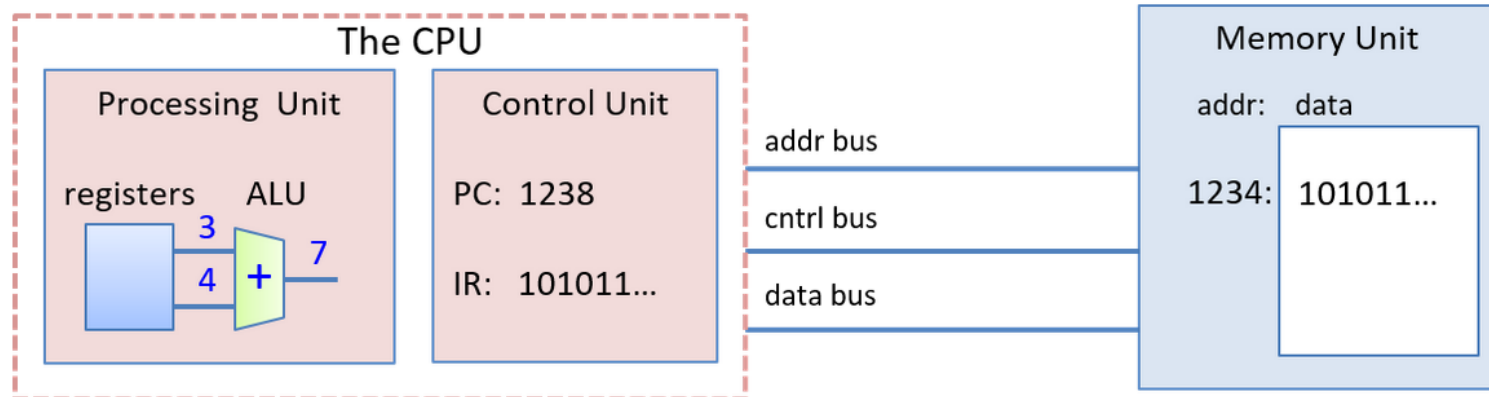


1. Fetch: Read instruction bits from memory at address in PC (1234), and store in IR

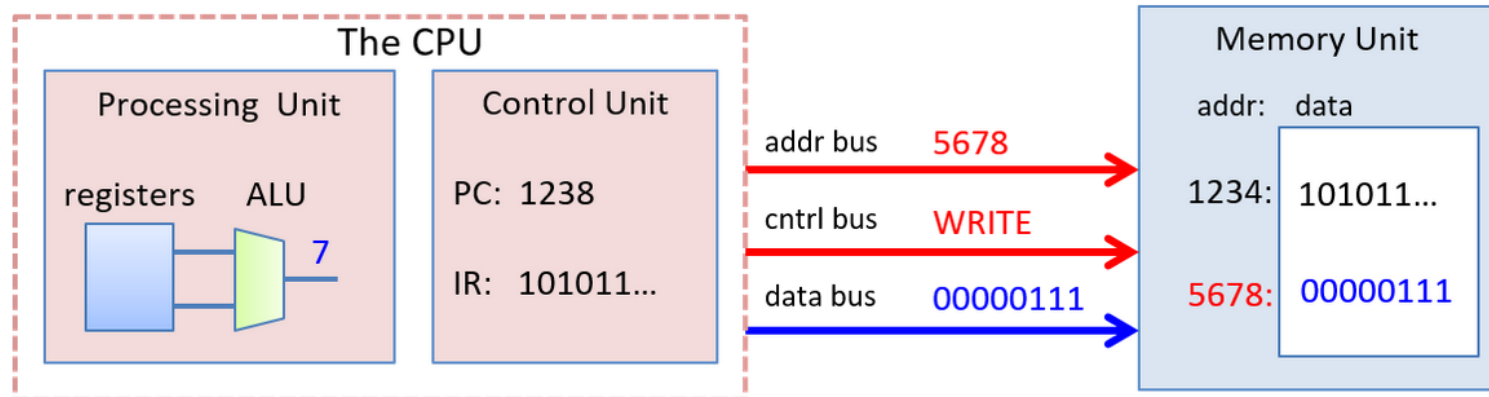


2. Decode: instruction bits in IR encode which registers store operands & the ALU operation

Example: Adding two numbers



3. Execute: ALU performs instruction operation (+) on operands (3,4) to compute result (7)

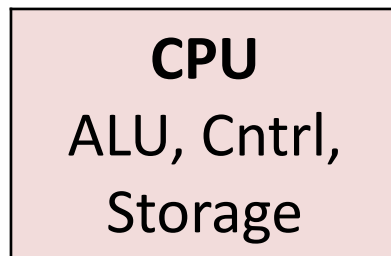


4. Store: the control unit stores the ALU result (7, binary 00000111) to memory

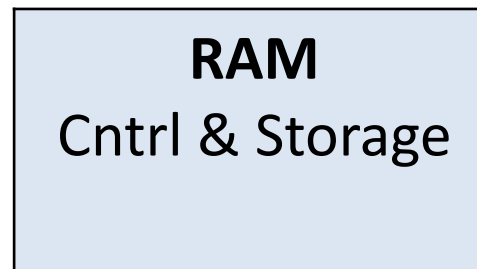
Digital Computers

- All input & output are discrete and binary
 - data, instructions, control signals (0: no voltage, 1: voltage)
 - execution is driven by a clock (will discuss later)
 - time is discrete: time 1, time 2, time 3, ...
- To run program, need different types of circuits

Circuits to
execute
program
instructions
that act on
program data



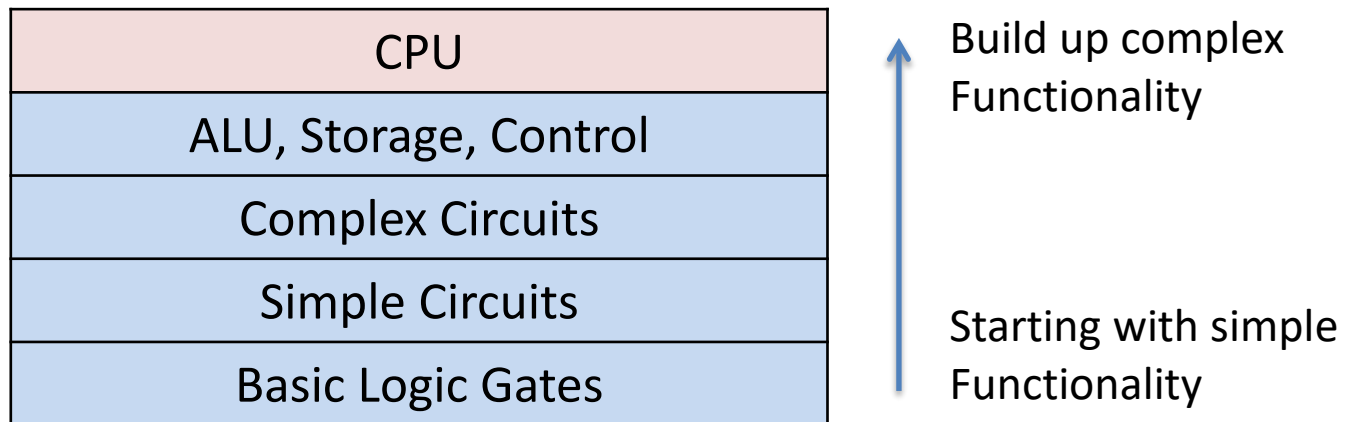
bus



Circuits to
store program
data and instructions
and support reading
and writing
addressable storage
locations

Building a CPU (model)

Levels of abstraction: start with very simple functionality, and add complexity



Logic Gates: Basic Building Blocks

Input: Boolean value(s) (high and low voltages for 1 and 0)

Output: Boolean value (0 or 1) result of boolean function

Always present, but may change when inputs change

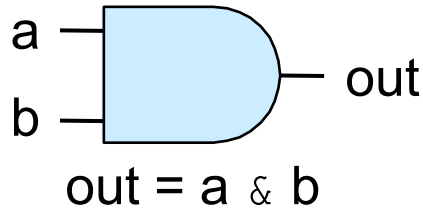
C bit-wise
operators:

& : AND

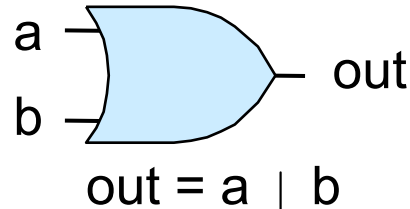
| : OR

~ : NOT

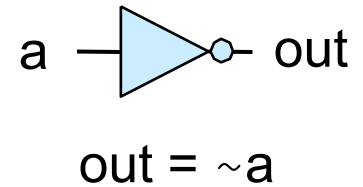
And



Or



Not



A	B	A & B	A B	~A
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

Review: C Bitwise Operators

Bit-wise operators: applied to bits of operand(s)

& (AND)

| (OR)

^(XOR)

~(NOT)

01010101	01101010	10101010	<u>~10101111</u>
& <u>00100001</u>	<u>10111011</u>	^ <u>01101001</u>	01010000
00000001	11111011	11000011	

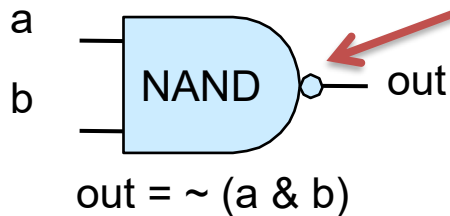
Evaluate Differently than C logical operators

```
char x=8, y=7;
```

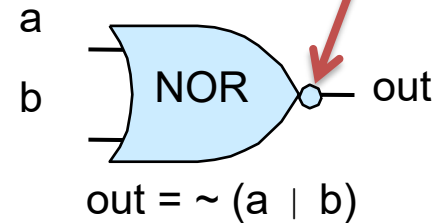
```
x & y // 8 bitwise AND 7 is 0
```

```
x && y // 8 logical AND 7 is a non-zero value (evals to true)
```

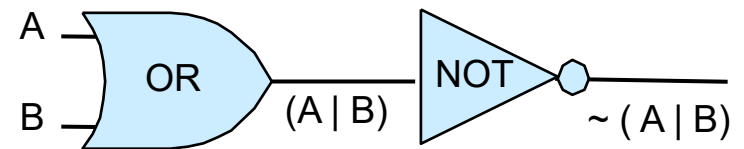

More Logic Gates



Note the circle on the output.
This means "negate it."



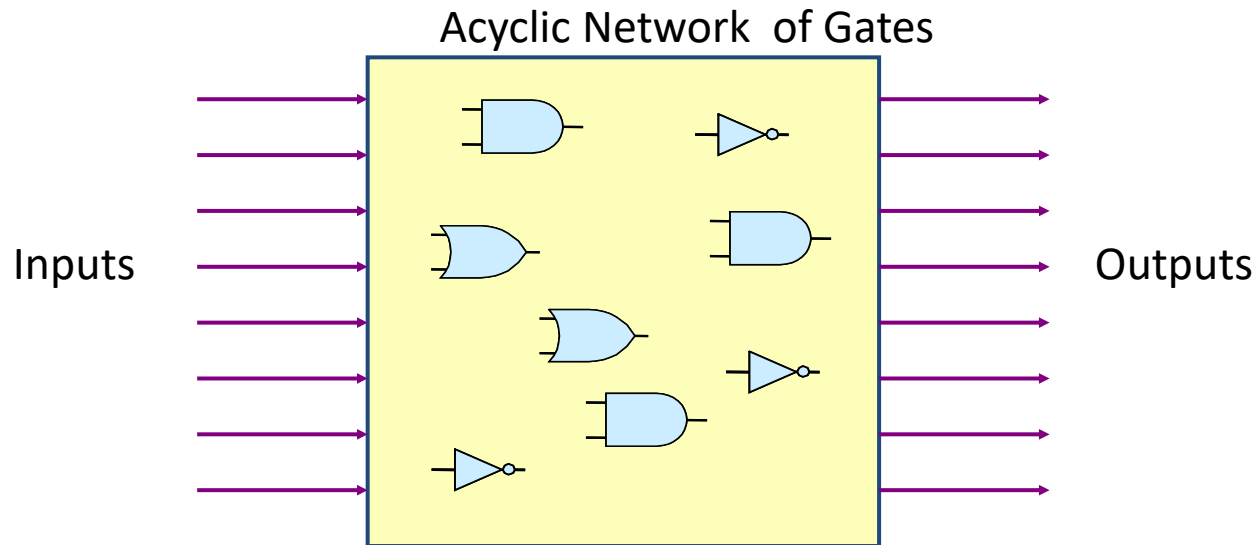
NOR ==



A	B	A	NAND	B	A	NOR	B
0	0		1			1	
0	1		1			0	
1	0		1			0	
1	1		0			0	

Combinatorial Logic Circuits

Idea: Combine logic circuits together to implement higher-level functionality

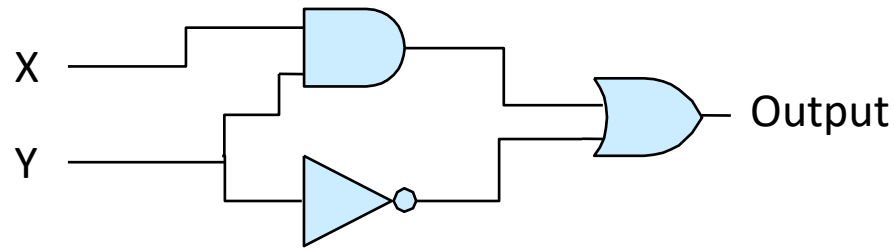
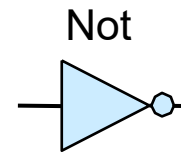
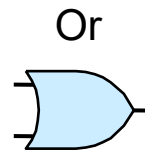
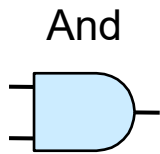


Outputs are boolean functions of inputs

Outputs continuously respond to changes to inputs

Use this new functionality as a building block for even higher level functionality (Abstraction!)

Exercise: Draw the truth table for this circuit



X	Y	Output

Exercise: Build a circuit that implements XOR using AND, OR, and NOT

Step 1: Construct the logic table

Exercise: Build a circuit that implements XOR using AND, OR, and NOT

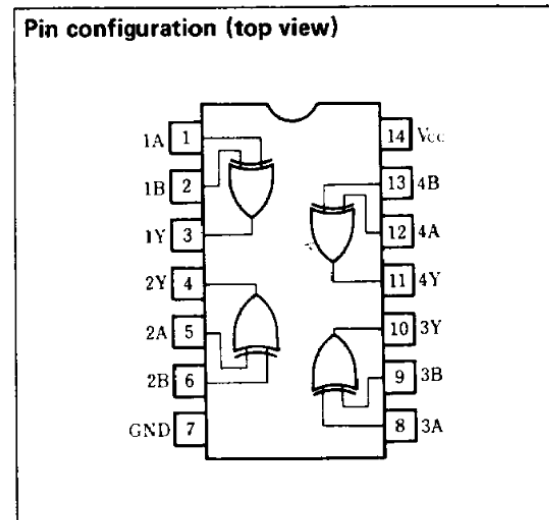
Step 2: Find an expression for $X \oplus Y$ using binary operations

Exercise: Build a circuit that implements XOR using AND, OR, and NOT

Step 3: Translate the expression to a circuit

Abstractions in action

Treat XOR Circuit as a building block for other circuits!



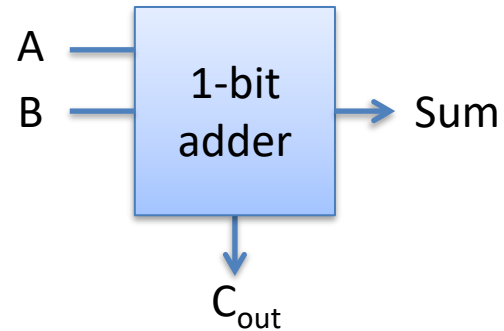
Arithmetic logic unit (ALU)

The **ALU** performs addition, subtraction, or, add, etc.

- Adder
- Subtractor
- Multiplication / Division (we are not covering these)
- Bit-wise AND, OR, NOT, XOR
- Bit shift operations
- Goal: reuse hardware components whenever possible

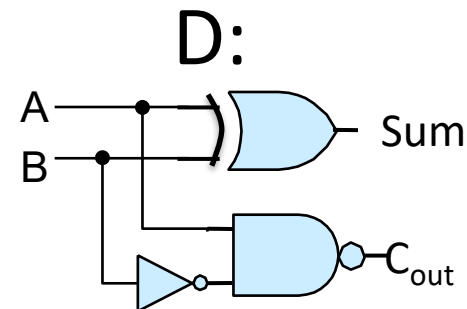
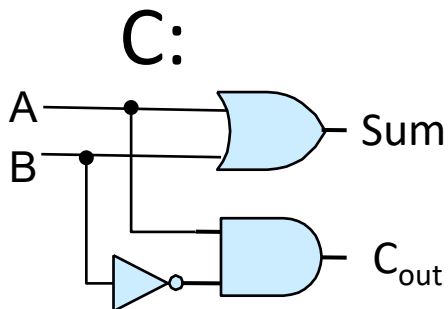
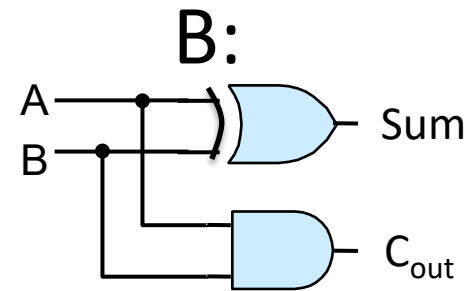
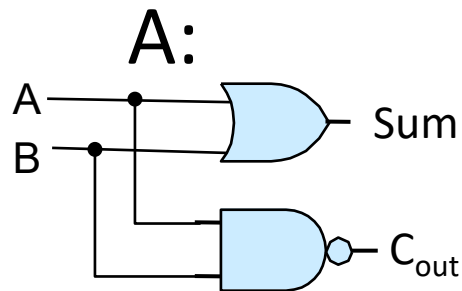
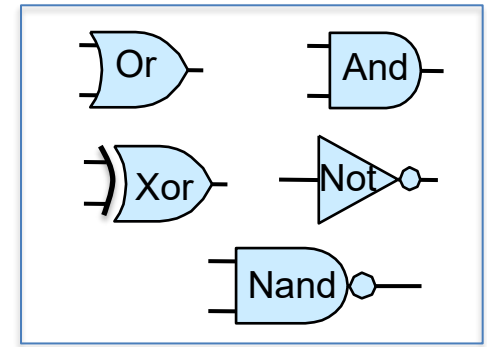
ALU: a 1-bit adder circuit

- 1 bit adder: $A+B$
- Two outputs:
 1. Obvious one: the sum
 2. Other one: ??



A	B	Sum (A+B)	Cout
0	0		
0	1		
1	0		
1	1		

Which of these circuits is a one-bit adder?



More than one bit addition?

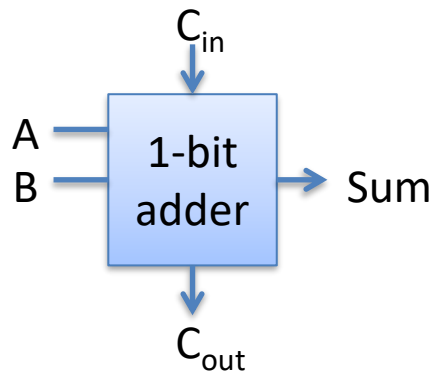
- When adding, sometimes have *carry in* too

$$\begin{array}{r} 000110\color{red}10 \\ + 000011\color{red}11 \\ \hline \end{array}$$

One-bit (full) adder

Need to include:

Carry-in & Carry-out



A	B	C _{in}	Sum	C _{out}
0	0	0	0	0
0	1	0	1	0
1	0	0	1	0
1	1	0	0	1
0	0	1	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1

- When is Sum 1?

$$\sim C_{in} \ \& \ (A \wedge B) \quad | \quad C_{in} \ \& \ \sim (A \wedge B) \quad == \quad (C_{in} \wedge (A \wedge B))$$

- When is C_{out} 1?

$$(A \ \& \ B) \quad | \quad ((A \wedge B) \ \& \ C_{in})$$

One-bit (full) adder

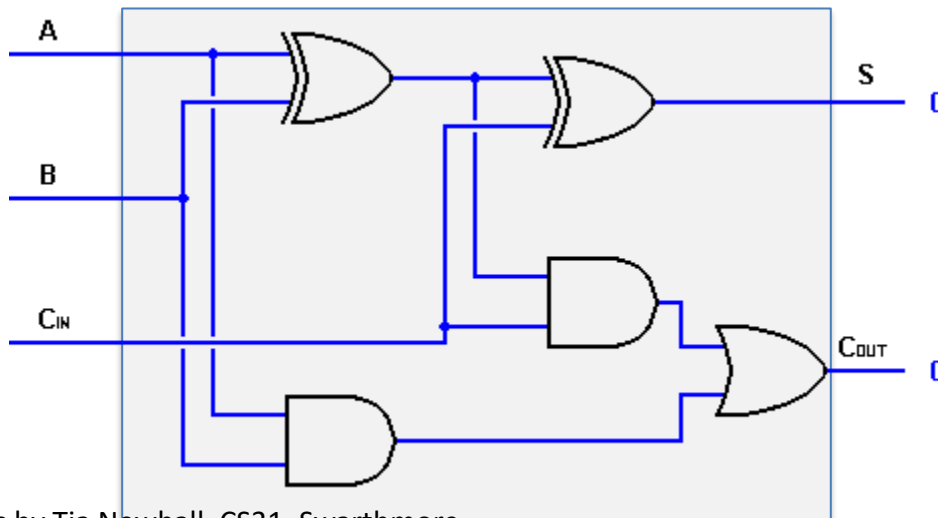
Need to include:

Carry-in & Carry-out

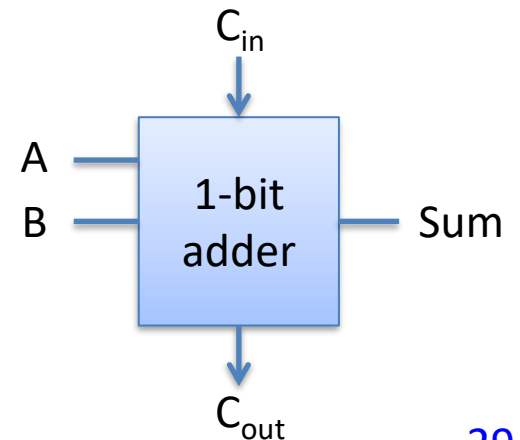
$$\text{Sum: } C_{in} \oplus (A \oplus B)$$

$$C_{out}: (A \& B) \mid ((A \oplus B) \& C_{in})$$

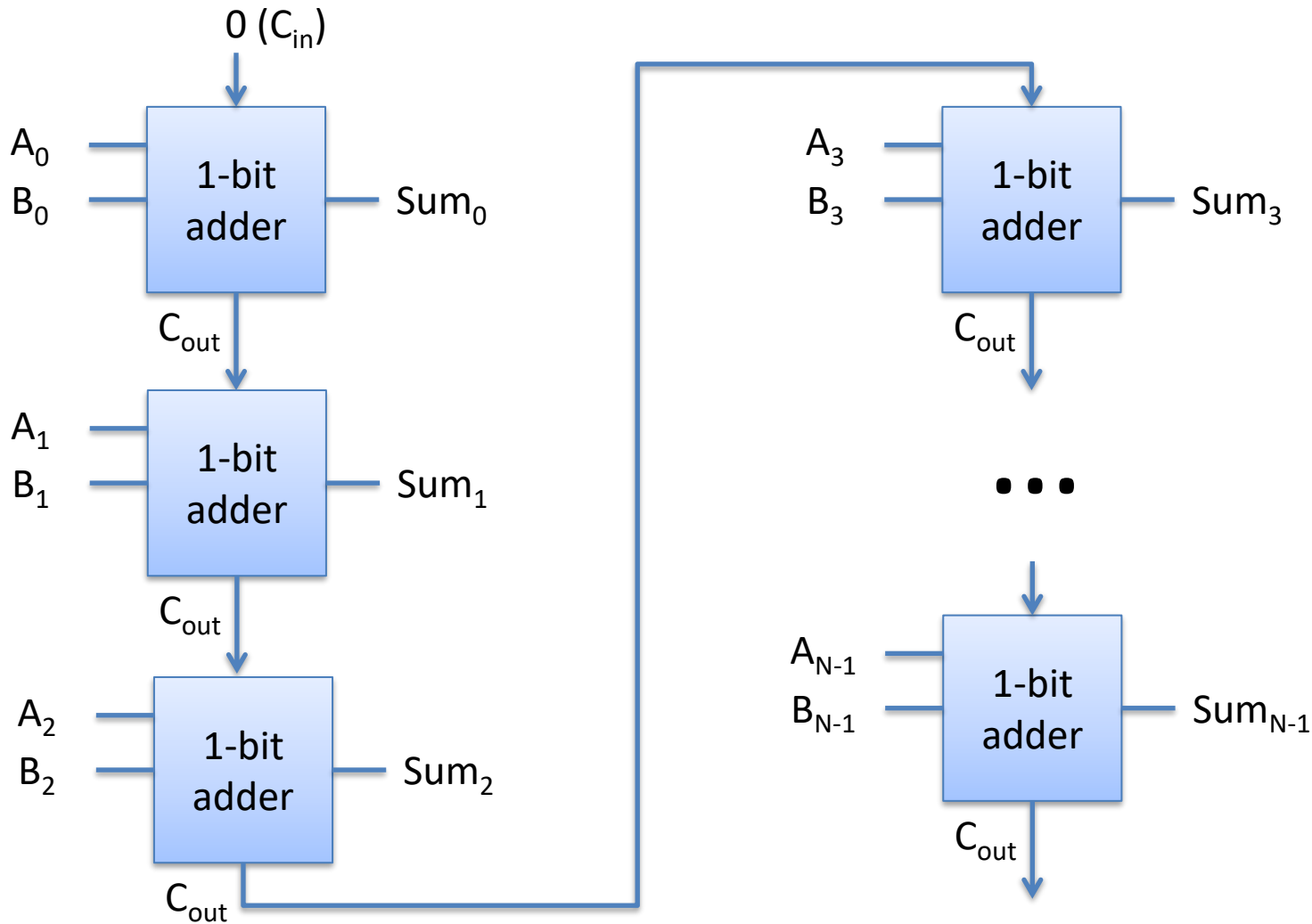
A	B	C _{in}	Sum	C _{out}
0	0	0	0	0
0	1	0	1	0
1	0	0	1	0
1	1	0	0	1
0	0	1	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1



=

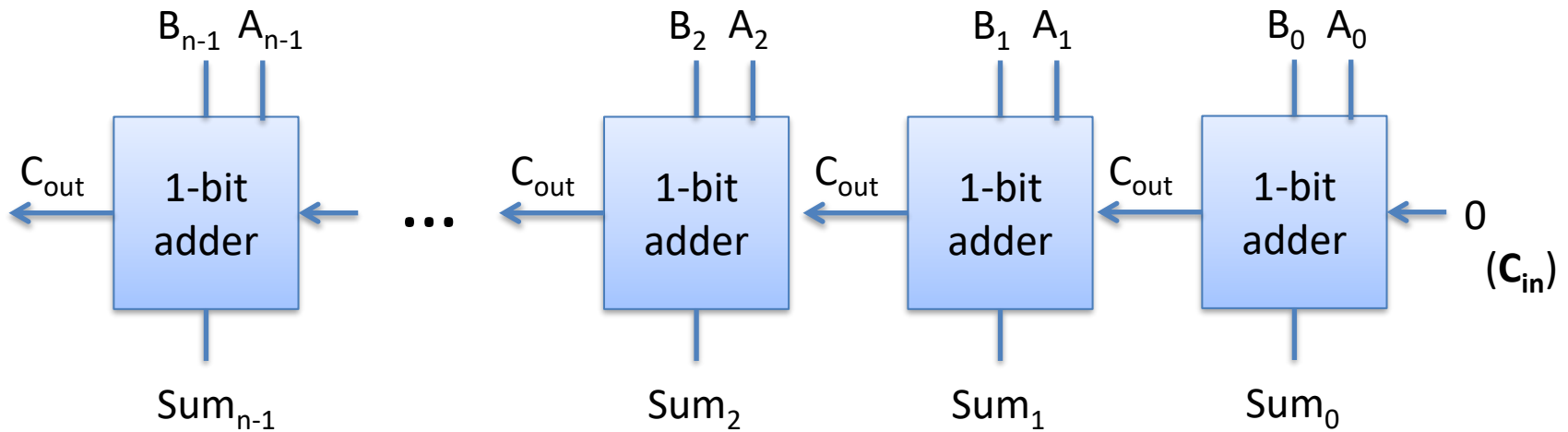


Multi-bit Adder (Ripple-carry Adder)



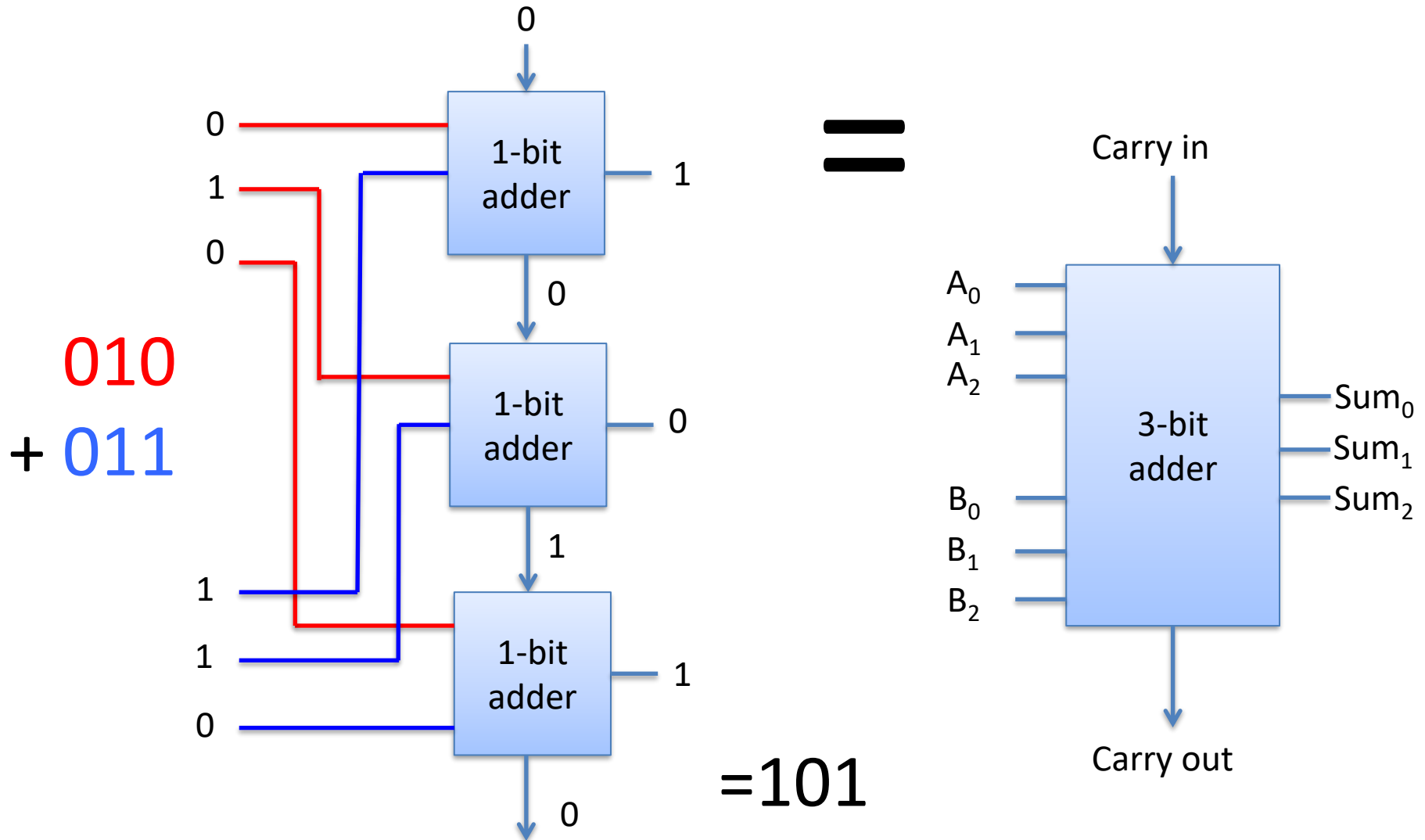
Multi-bit Adder (Ripple-carry Adder)

0	. . .	0	1	1	A
1	. . .	1	1	0	
					B

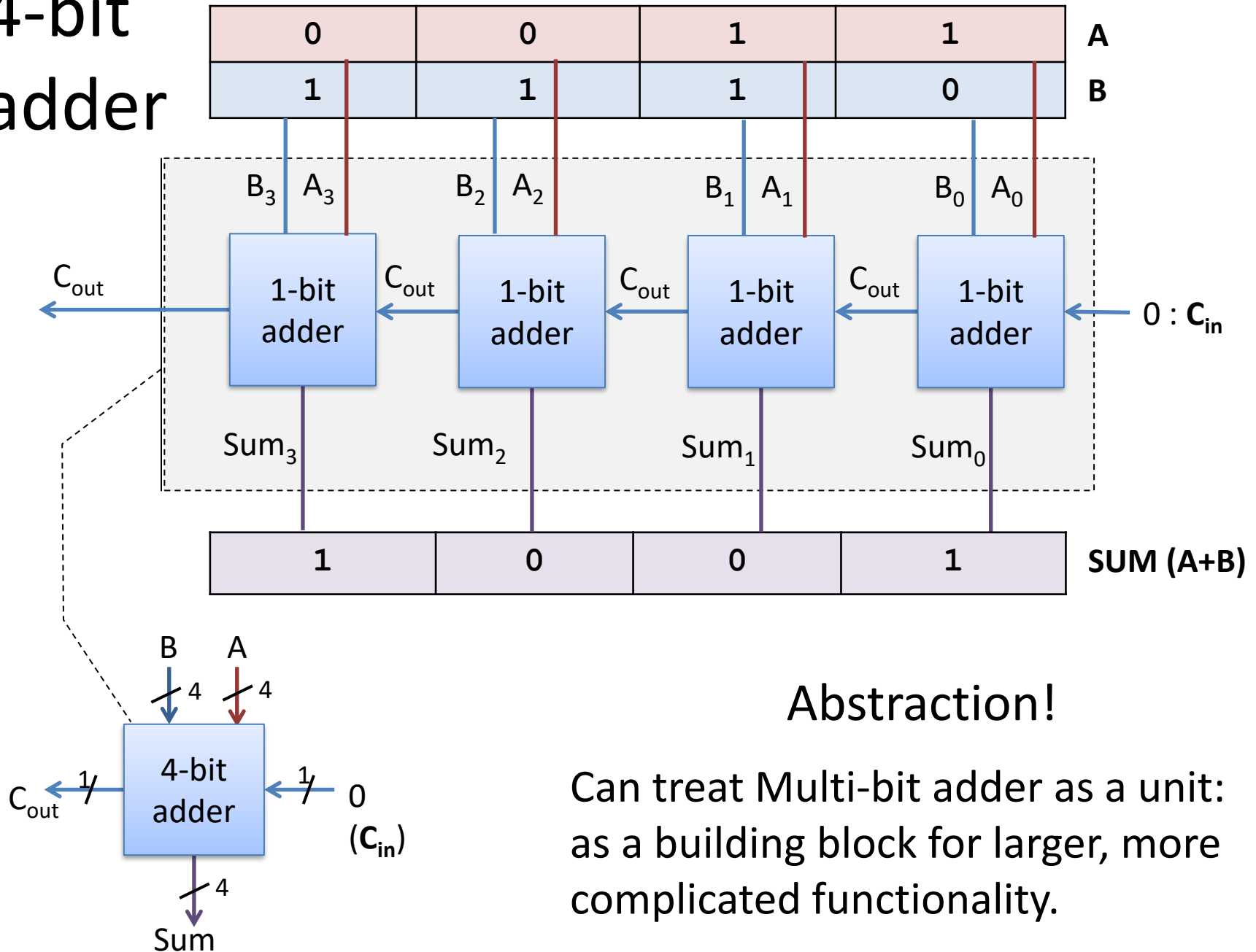


1	. . .	0	0	1	SUM
---	-------	---	---	---	-----

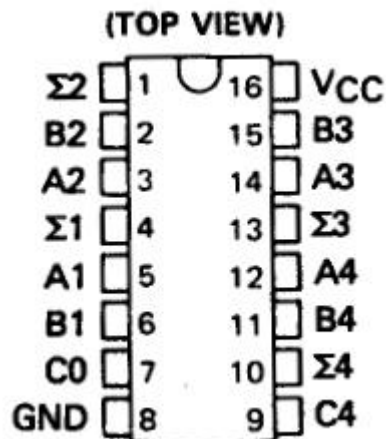
Three-bit Adder (Ripple-carry Adder)



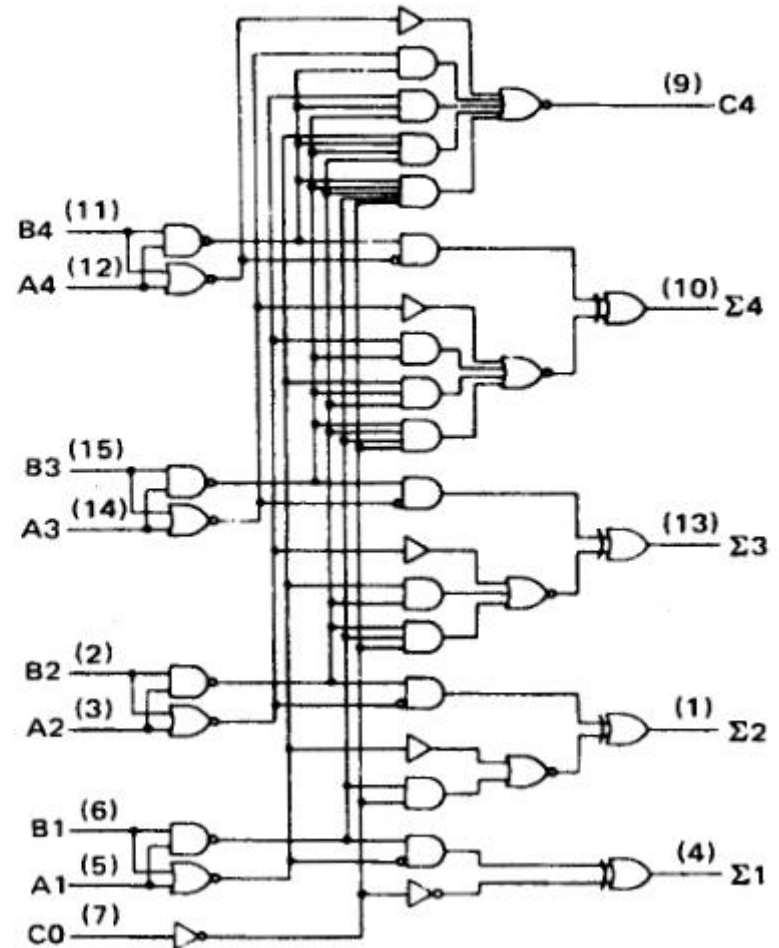
4-bit adder



4-bit adder real life example



logic diagram (positive logic)

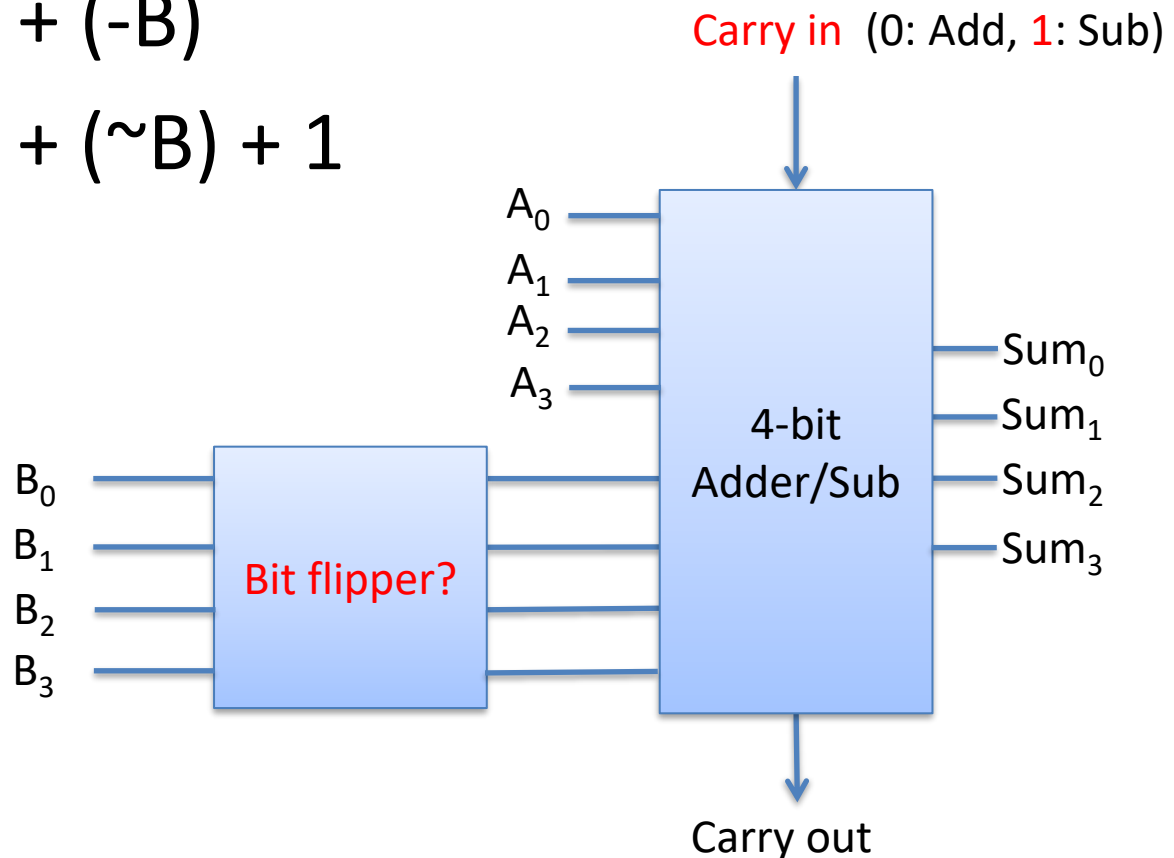


How to Implement Subtraction Circuit?

Q: what is relationship between Add & Sub?

$$A - B == A + (-B)$$

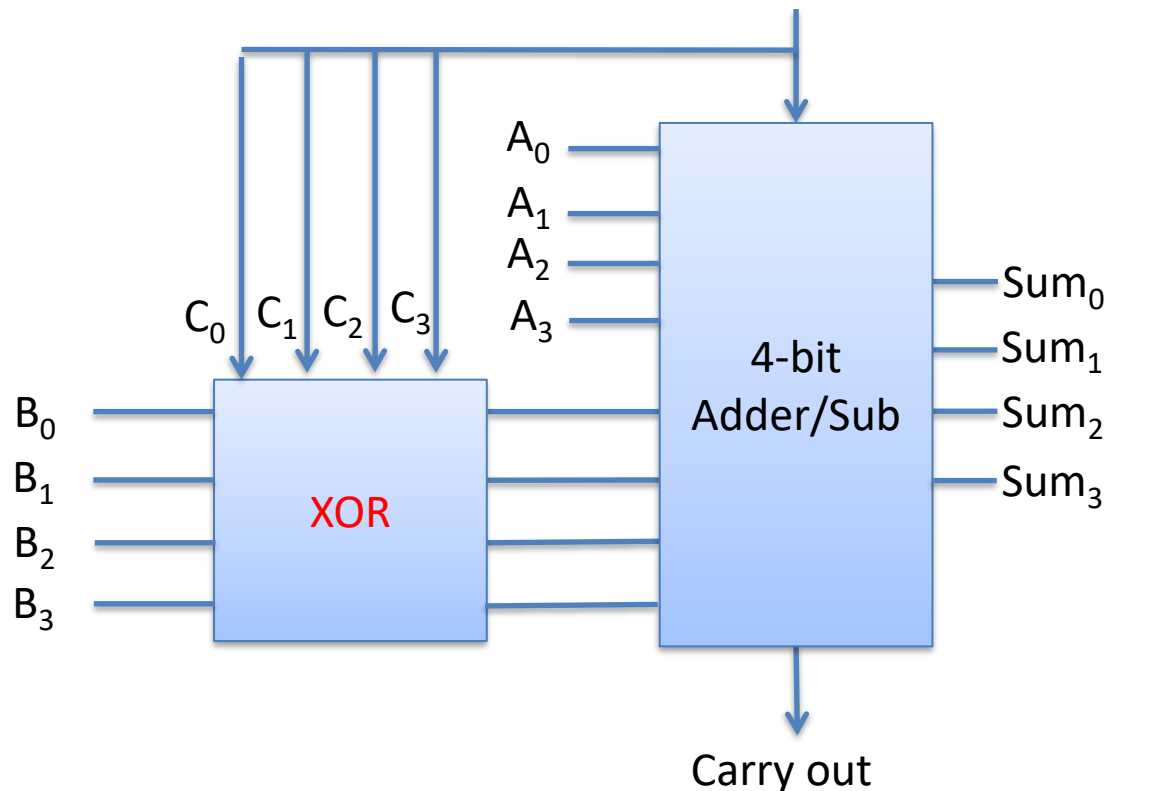
$$A - B == A + (\sim B) + 1$$



How to Implement Subtraction Circuit?

Q: what is relationship between Add & Sub?

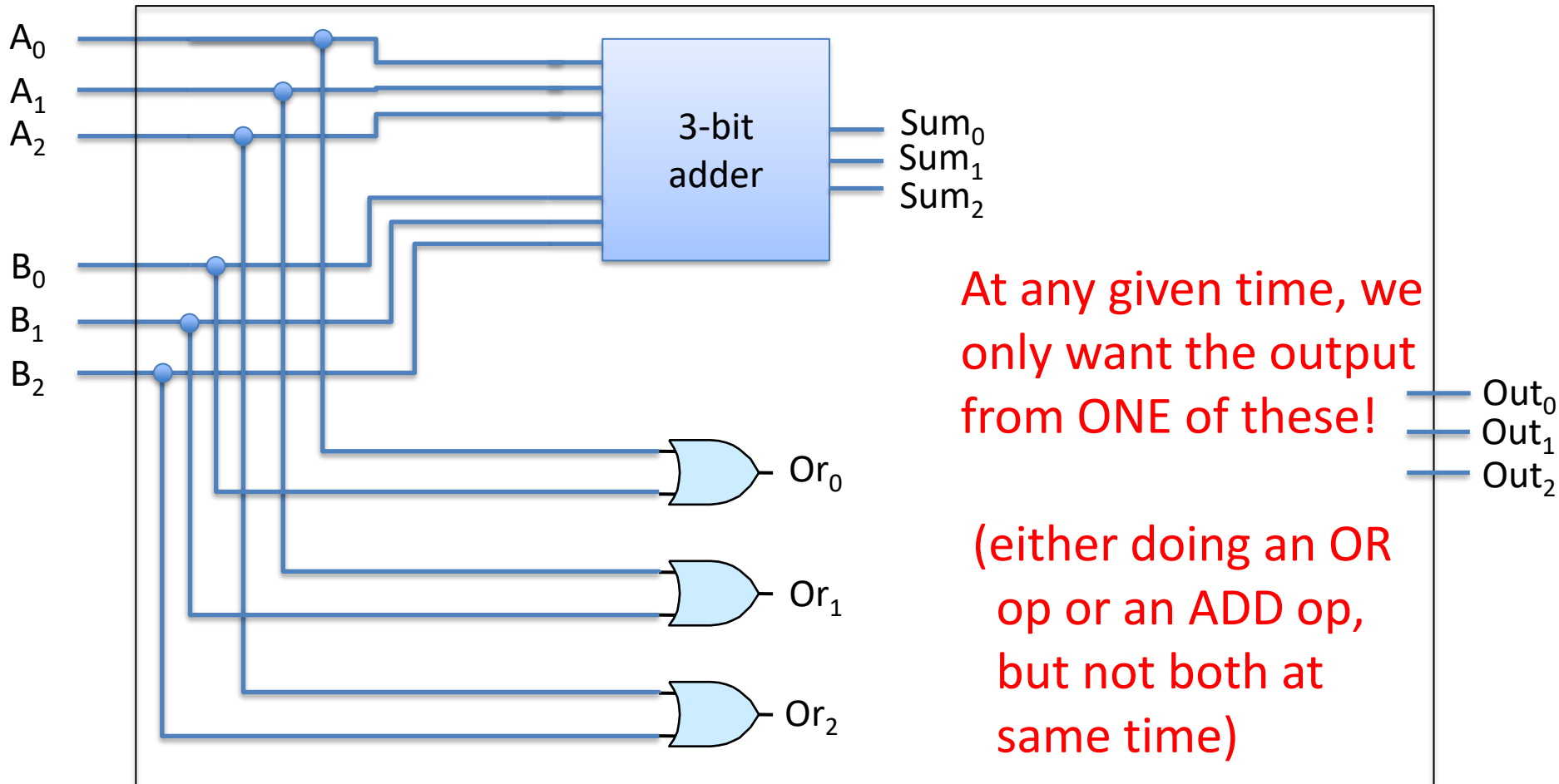
$$A - B == A + (\sim B) + 1$$



Simple 3-bit ALU: Add and bitwise OR

3-bit inputs

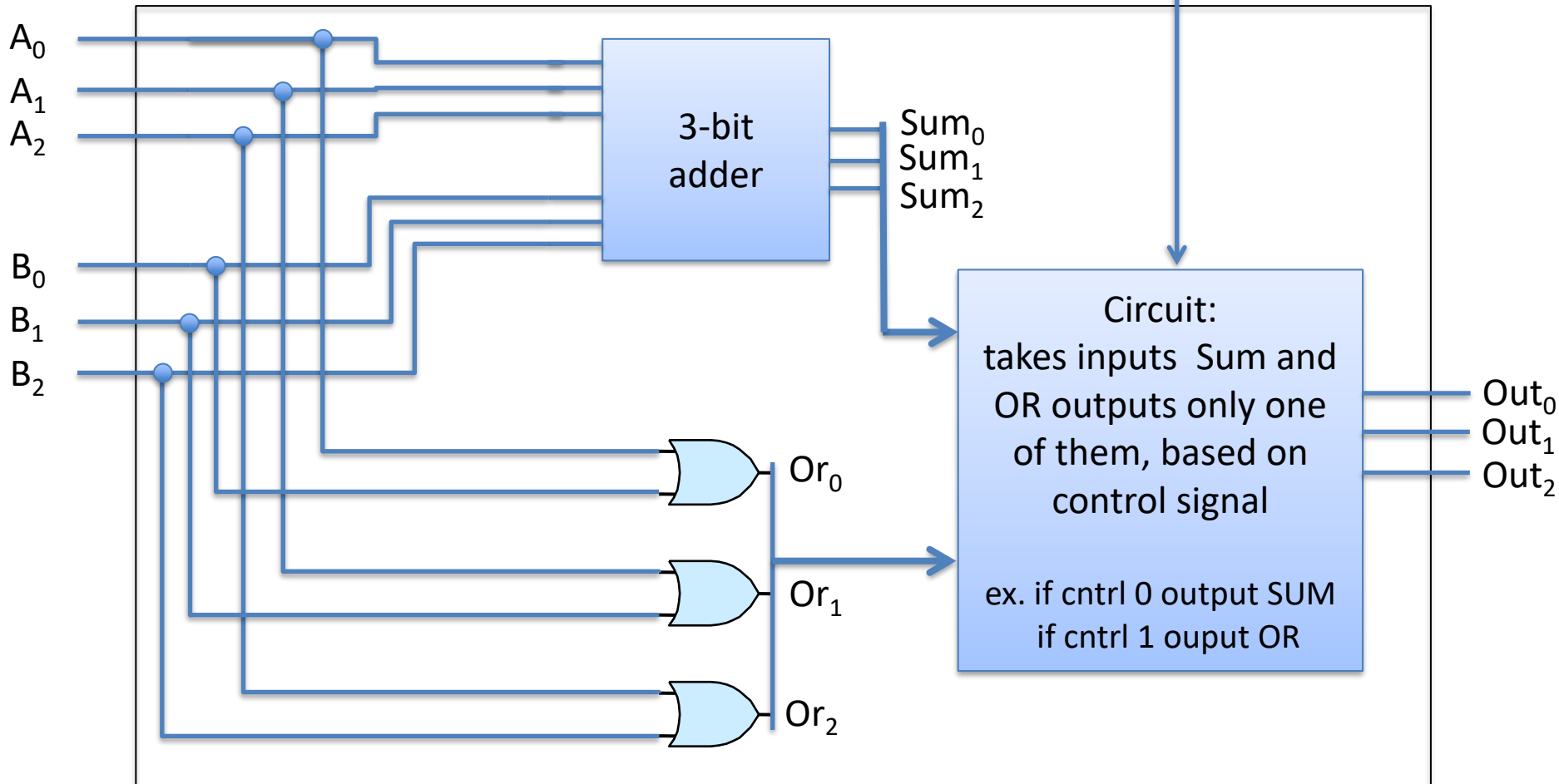
A and B:



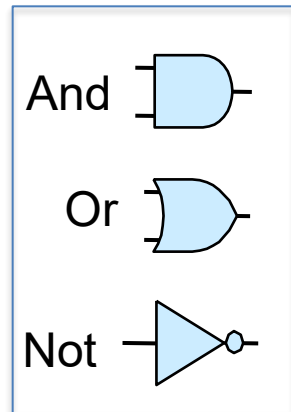
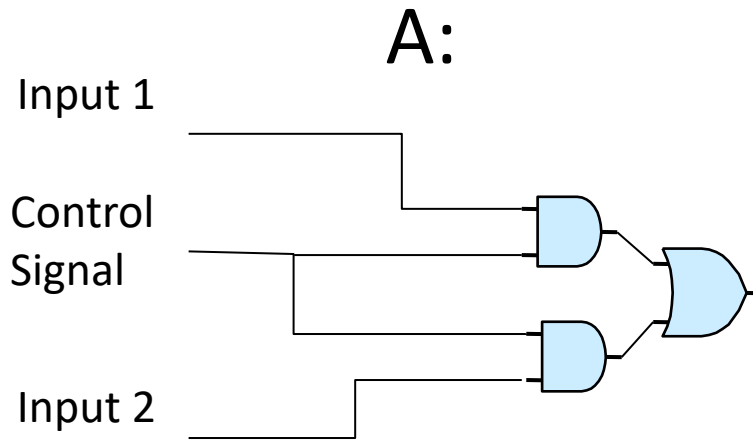
Simple 3-bit ALU: Add and bitwise OR

3-bit inputs
A and B:

Extra input: control signal to select Sum vs. OR

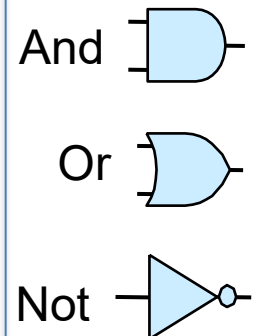
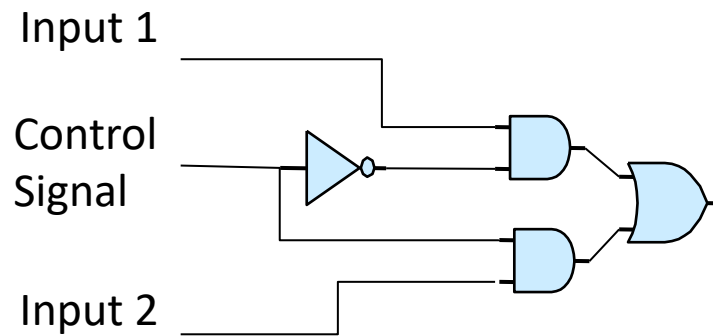


Draw the truth tables for this circuit



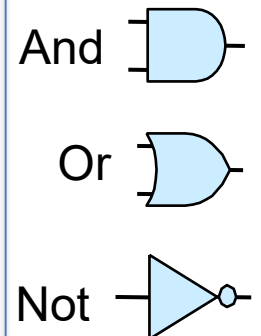
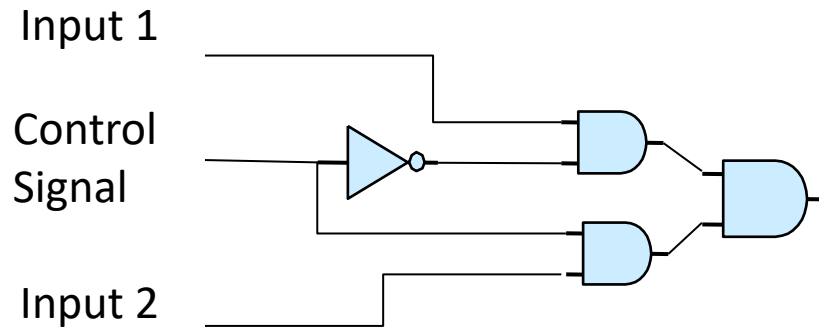
Draw the truth tables for this circuit

B:



Draw the truth tables for this circuit

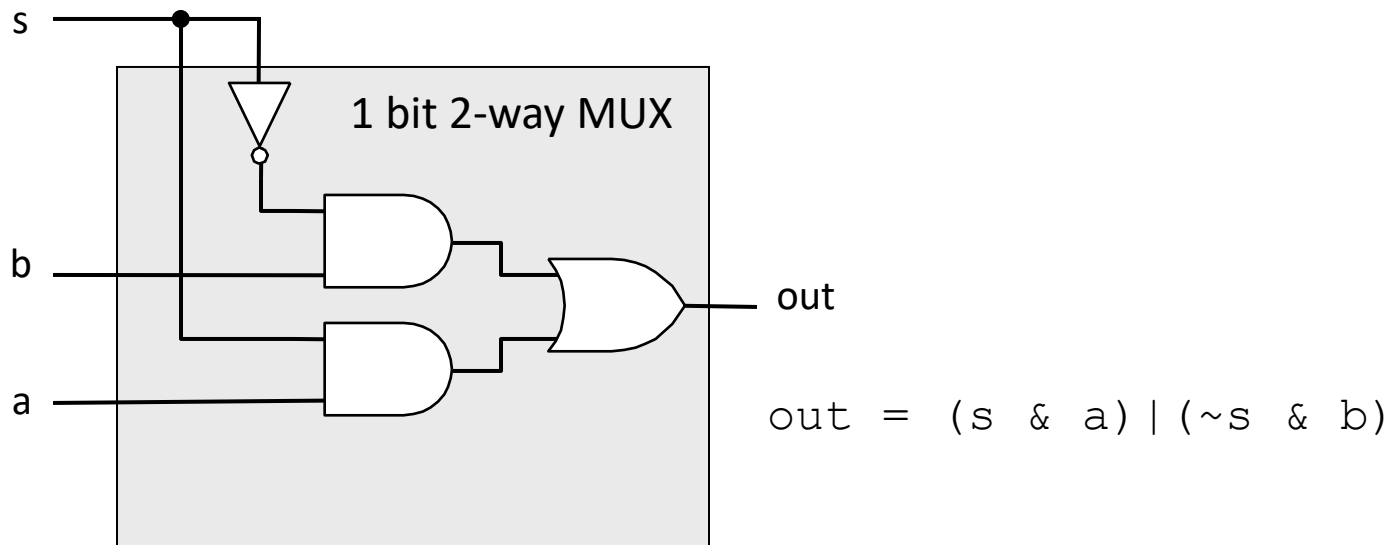
C:



Multiplexor: Chooses an input value

Inputs: 2^N data inputs, N signal bits

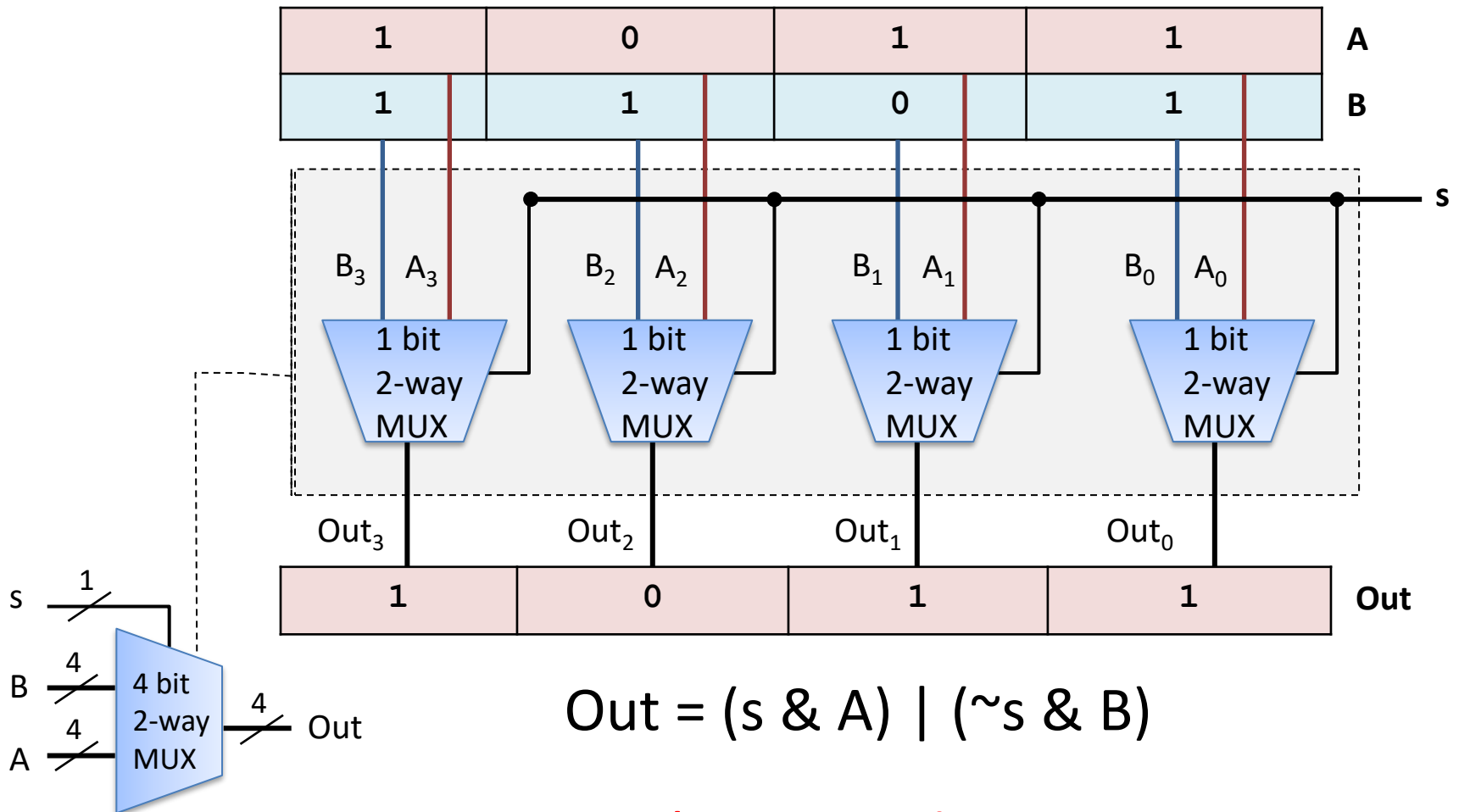
Output: is one of the 2^N input values



- Control signal s , chooses the input for output
 - When s is 1: choose a , when s is 0: choose b

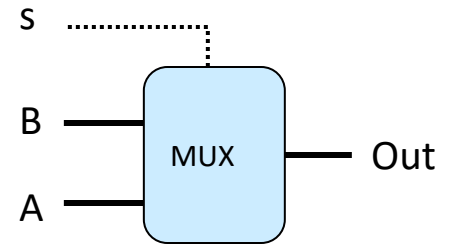
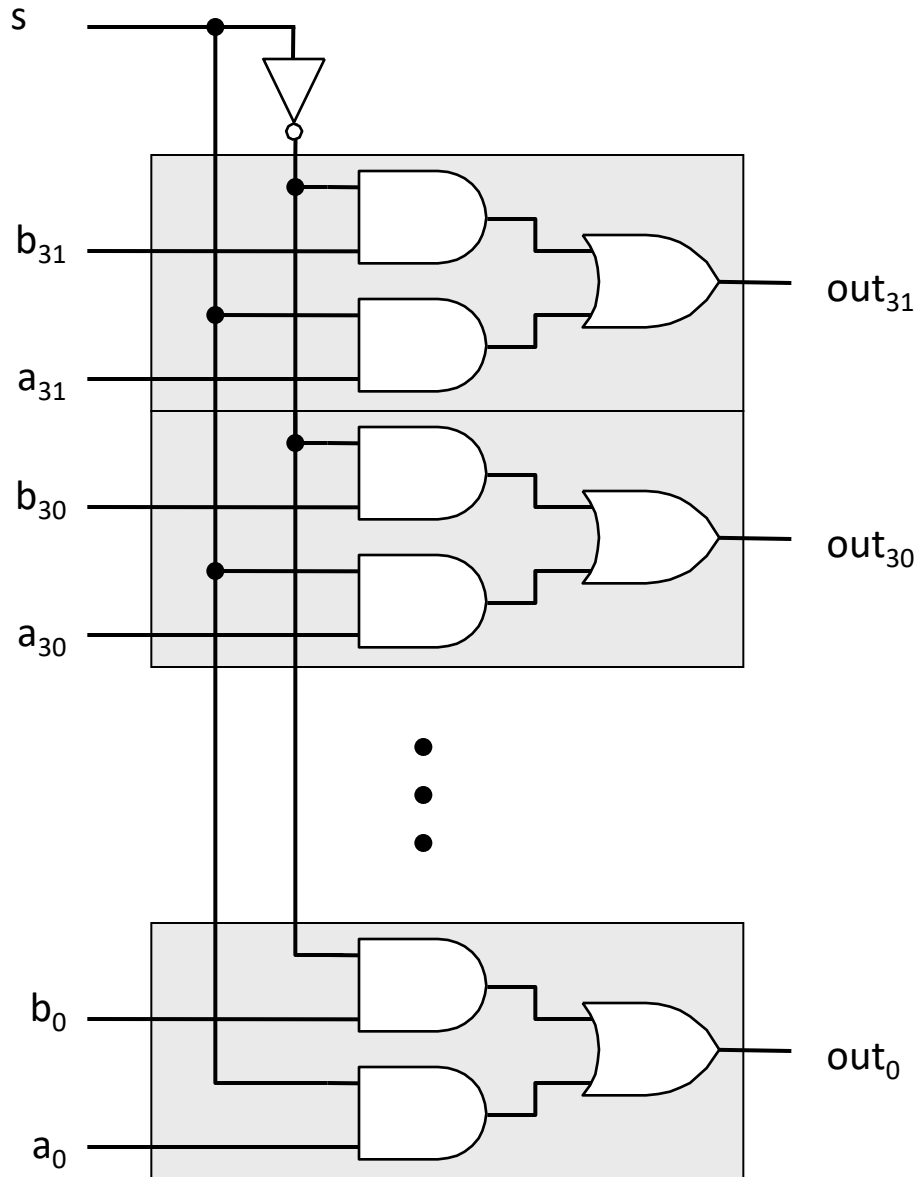
4-bit 2-way MUX 1-bit 2-way Muxes

Corresponding bits of A and B fed through a 1-bit MUX



Abstraction!

32 bit 2-Way Multiplexor



Input:

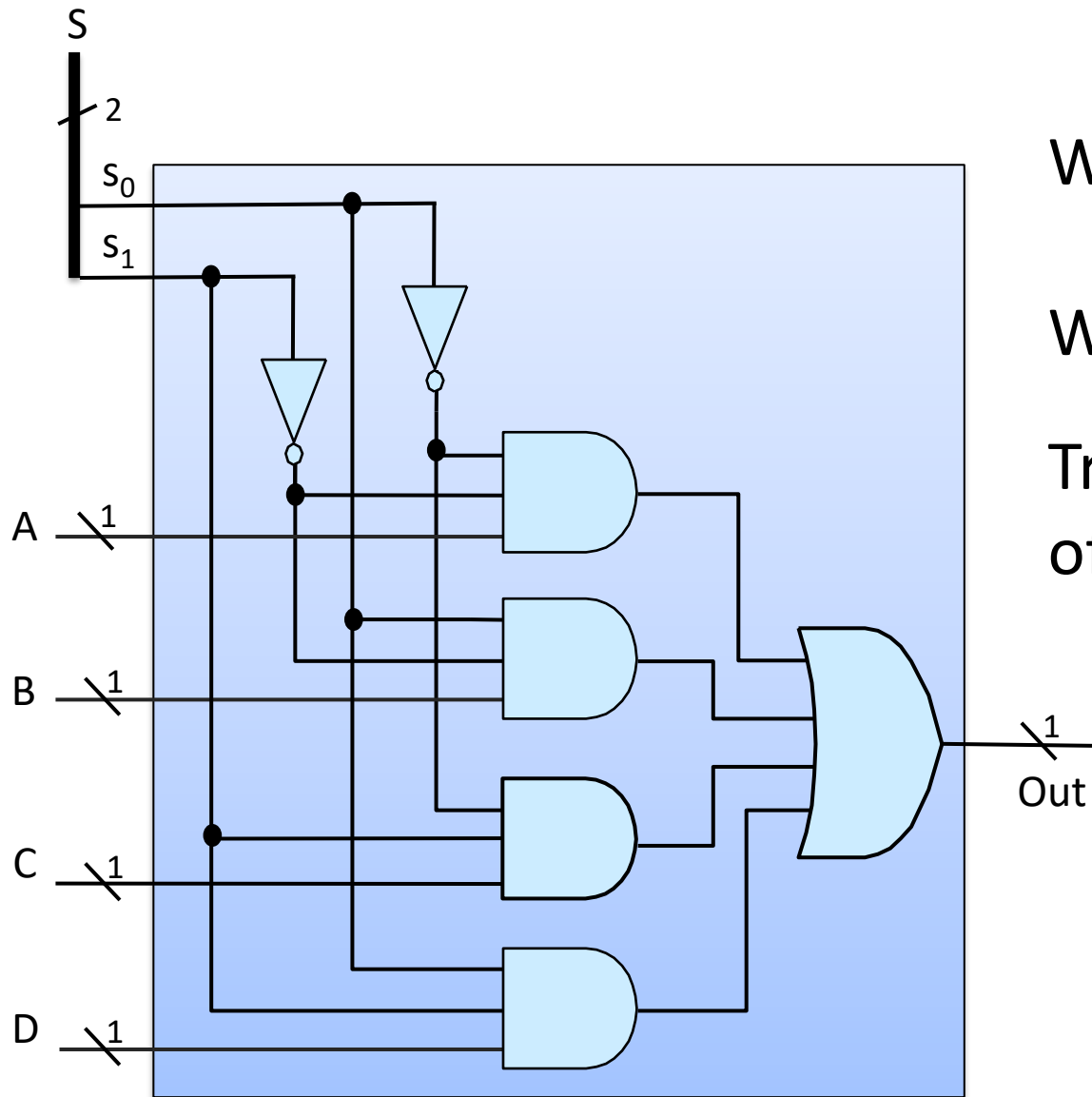
two 32 bit values (a , b)
one 1bit signal s

Each corresponding bit of
32 bit input values, fed
through 1 bit mux

Output: 1 of the 2 inputs

One 32 bit value
(either a or b)

What does this Circuit do?



What are inputs?

What does it output?

Try for different values of S (S is 2 bits of input)

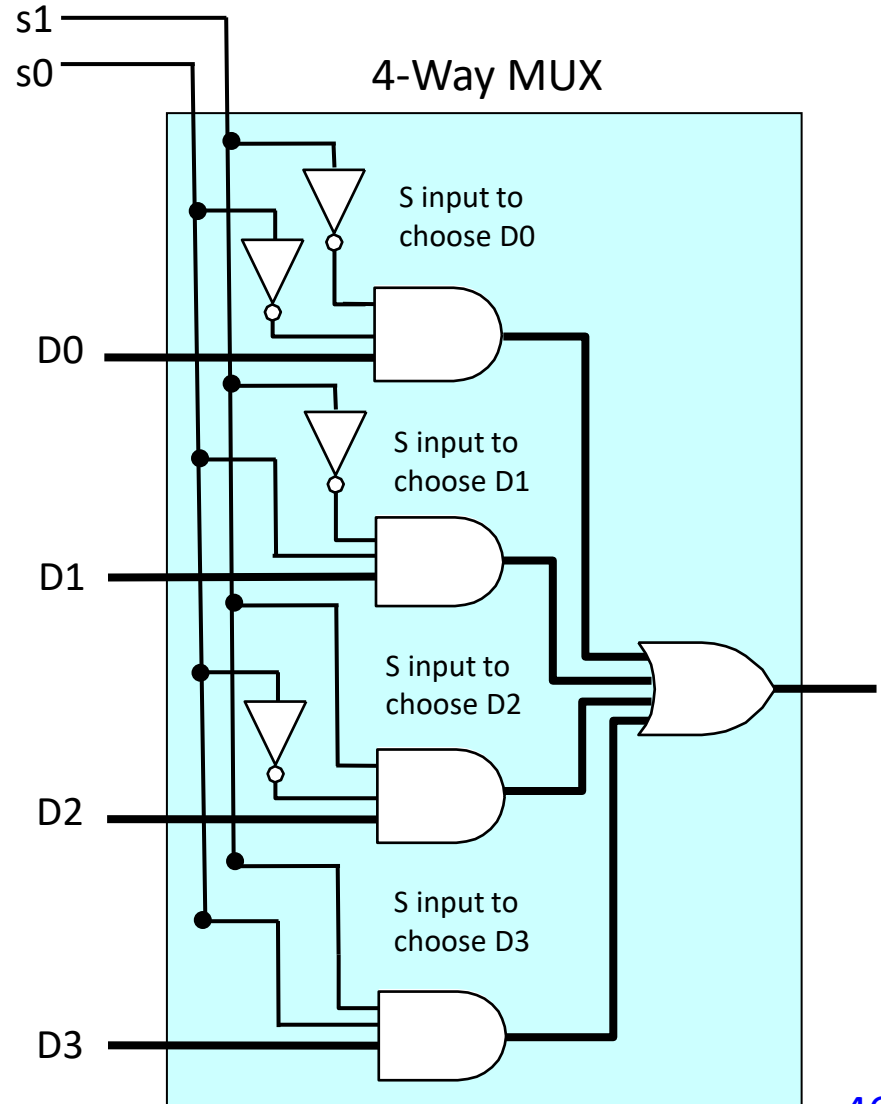
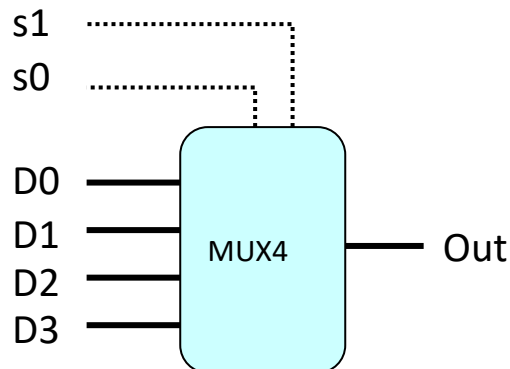
S (s_1s_0)	Out?
0 (00)	
1 (01)	
2 (10)	
3 (11)	

N-Way Multiplexor

Choose one of N inputs

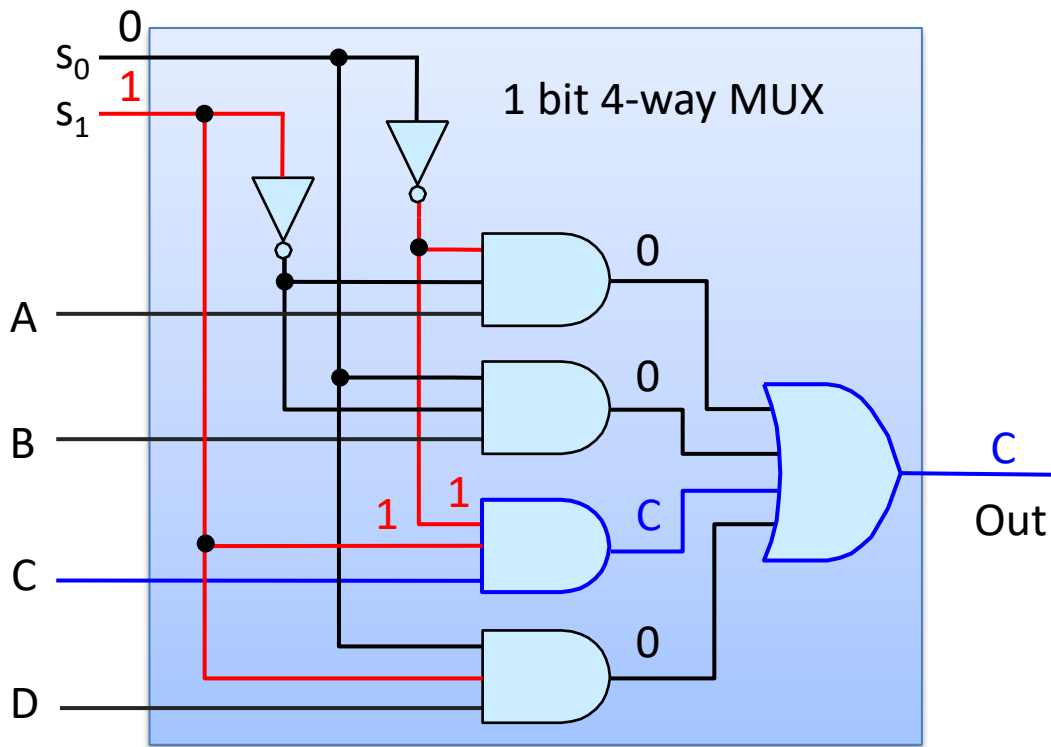
need $\log_2 N$ select bits

s1	s0	choose
0	0	D0
0	1	D1
1	0	D2
1	1	D3

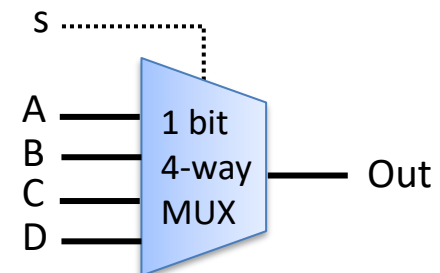


Example, 1-bit 4-way MUX

When select input is 2 (0b10): C chosen as output



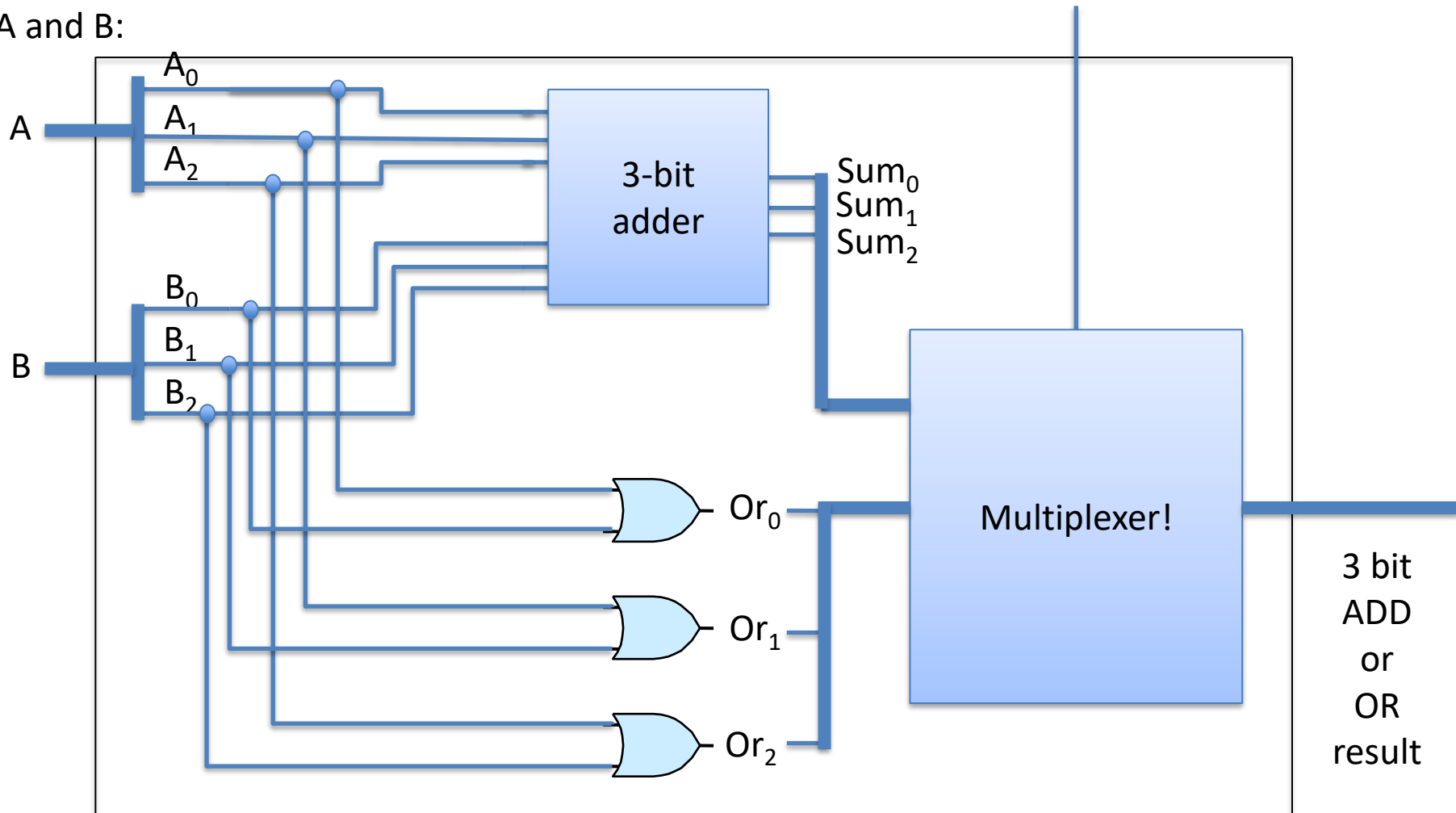
S	Out
0	A
1	B
2	C
3	D



Simple 3-bit ALU: Add and bitwise OR

3-bit inputs
A and B:

Extra input: control signal to select Sum vs. OR



Where do Select bits come from?

- Encoded in the bits of the CPU instruction to execute in IR:

IR:

10010011

- Instruction encodes information about
 - The operation to perform (OP): **selects the operation**
 - Sometimes operands for the operation (ex. val1, val2)
 - Sometimes destination for result

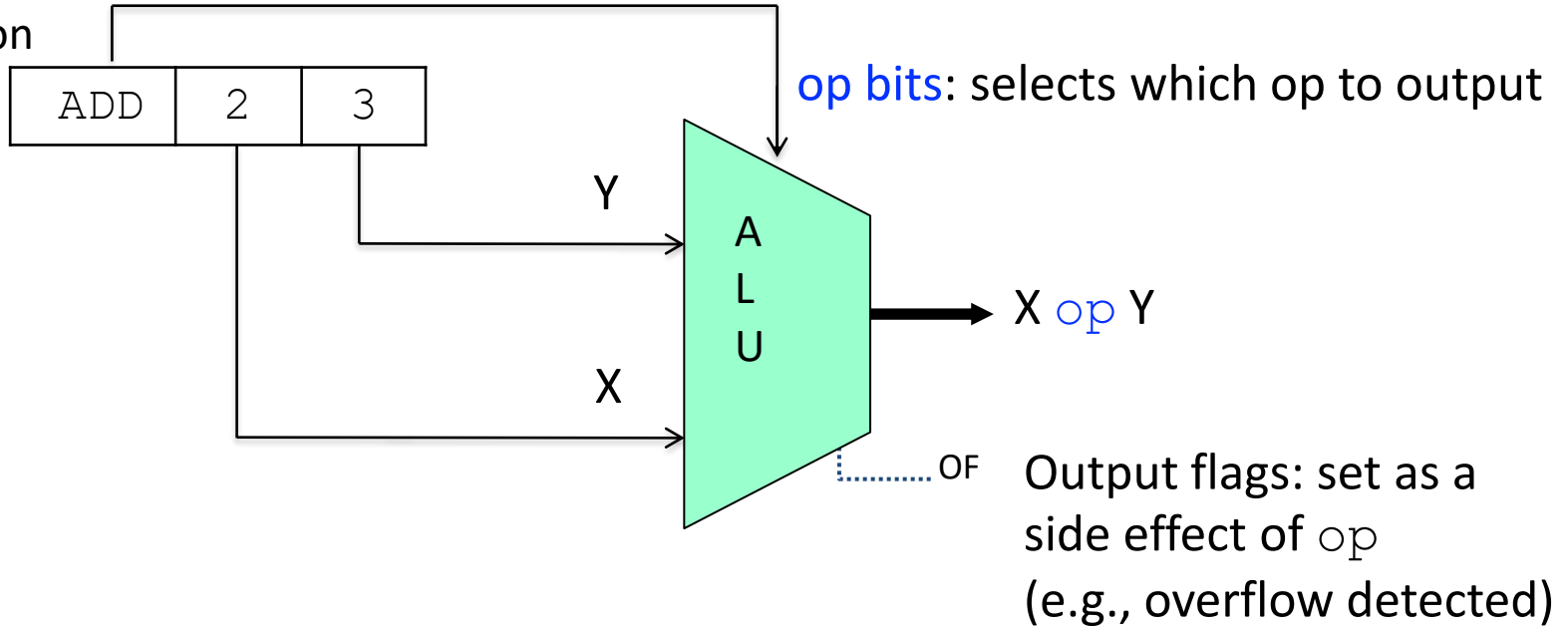
IR:	<table><tr><td>10</td><td>010</td><td>011</td></tr></table>			10	010	011	
10	010	011					
encodes:	<table><tr><td>OP</td><td>val1</td><td>val2</td></tr></table>	OP	val1	val2	compute: 2 + 3		
OP	val1	val2					
e.g.:	<table><tr><td>ADD</td><td>2</td><td>3</td></tr></table>	ADD	2	3			
ADD	2	3					

Summary: CPU so far

bits of CPU

Instruction

(in IR):



- Arithmetic and logic circuits: ADD, SUB, NOT, ...
- Control circuits: MUX use op bits to select output
- Circuits around ALU:
 - Select input values X and Y from instruction or register
 - Select op bits from instruction to feed into ALU
 - Feed output somewhere

Summary: Building a CPU

Three main classifications of HW circuits:

1. ALU: implement arithmetic & logic functionality
(ex) adder to add two values together
2. Storage: to store binary values
(ex) Register File: set of CPU registers
3. Control: support/coordinate instruction execution
(ex) fetch the next instruction to execute

Building a CPU: Storage

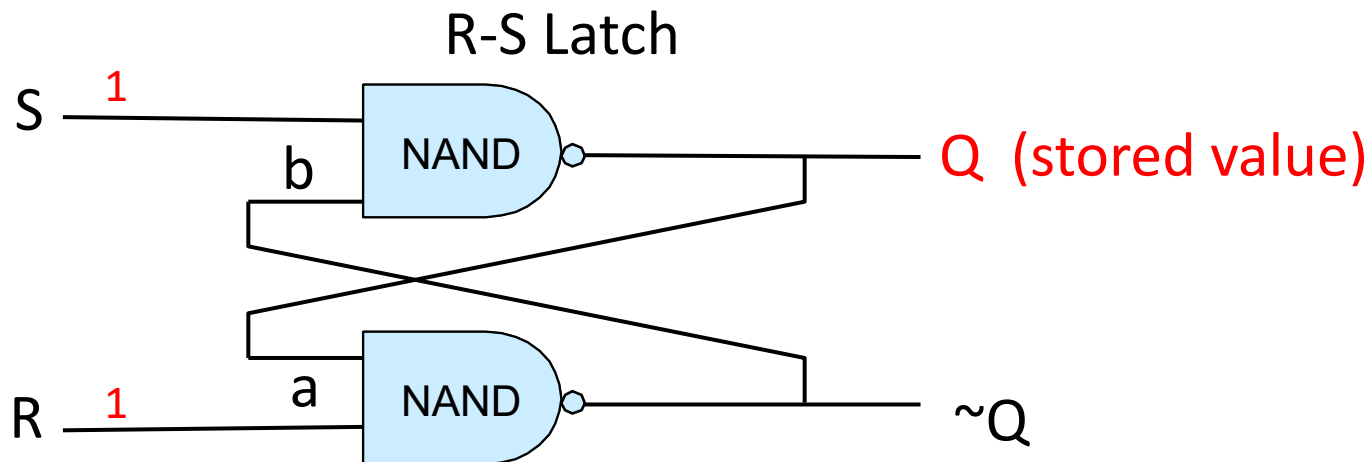
Goal: Give the CPU a “scratch space” to perform calculations and keep track of the state its in.

Baby steps:

- Store a 0 or 1
- Retrieve the 0 or 1 value on demand (read)
- Set the 0 or 1 value on demand (write)

R-S Latch: Stores Value Q

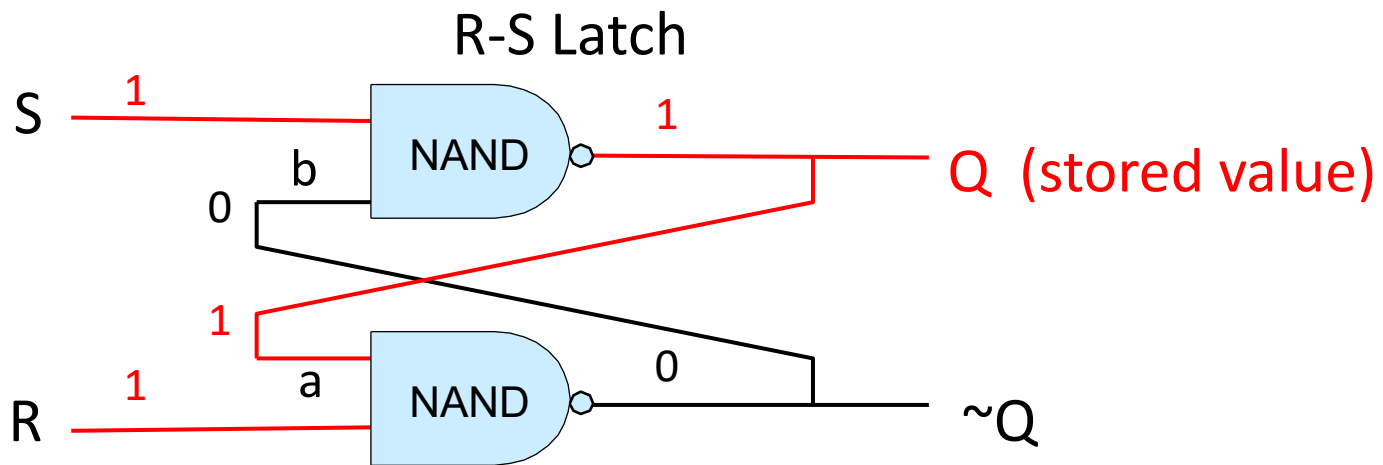
- To store a value need a circuit with a Cycle
- Value is **Stored** when R and S **are both 1**
 - Stored value, **Q**, is stable in this circuit
 - External control circuitry ensures R and S are 1



ex. latch stores 0 if a is 0: $R = 1 \rightarrow b \text{ is } 1, S = 1 \text{ and } b = 1 \rightarrow Q \text{ is } 0$

Latch Stores value when R and S both 1

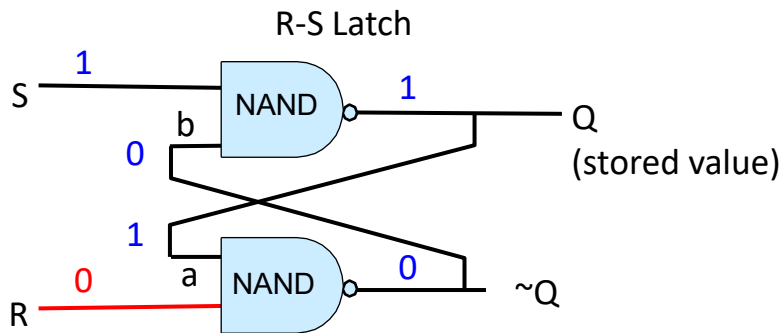
(ex) if a is 1: $R = 1 \rightarrow b \text{ is } 0$, $S = 1$ and $b = 0 \rightarrow Q \text{ is } 1$, and a is 1 ...



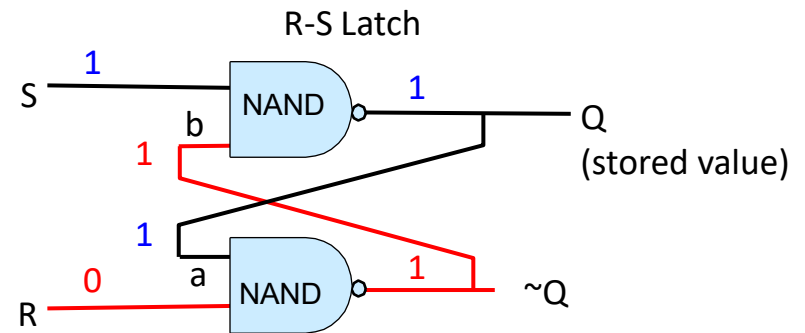
The latch stores 1

Ex. to write (and store) 0 into Latch

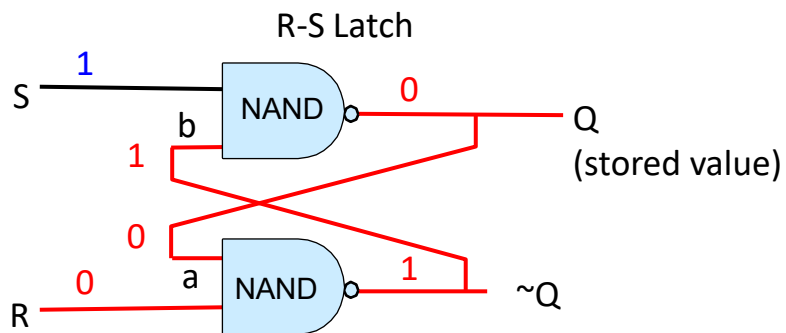
- Set R to 0 momentarily (S stays at 1)



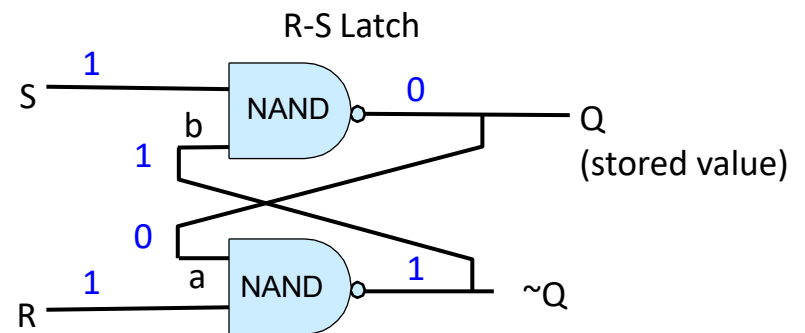
A. Set R to 0 to store 0



B. Changes lower NAND output to 1



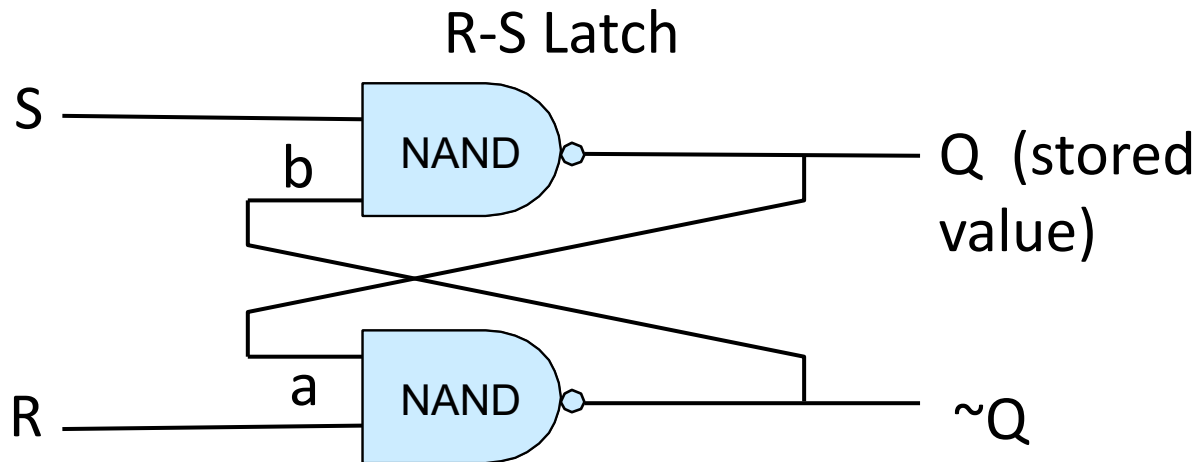
C. Changes upper NAND output to 0



D. R-S Latch Now Stores 0
(R can be set back to 1 and still stores 0)

R-S Latch: Stores Value Q

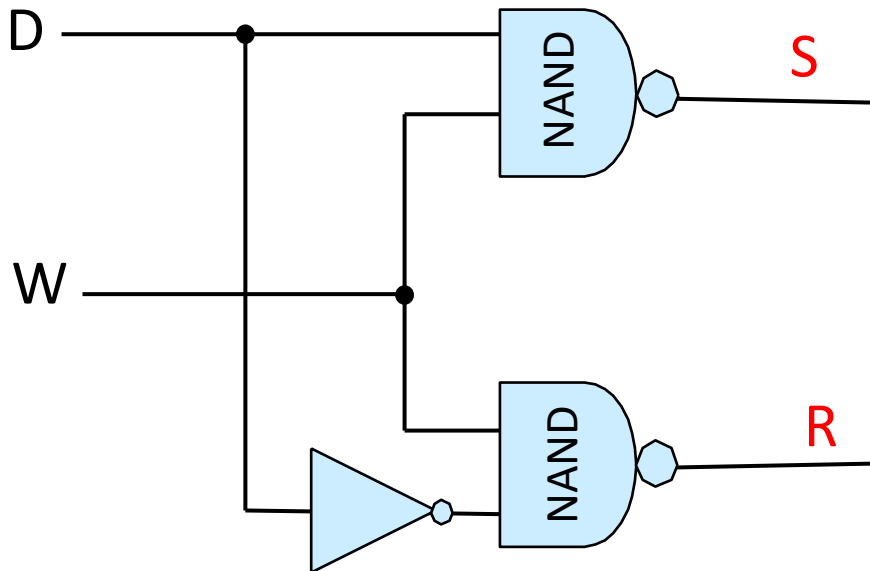
- When R and S are both 1: Store a value
R and S are never both simultaneously 0



if a is 1: $R = 1 \rightarrow b$ is 0, $S = 1$ and $b = 0 \rightarrow Q$ is 1: **latch stores 1**
if a is 0: $R = 1 \rightarrow b$ is 1, $S = 1$ and $b = 1 \rightarrow Q$ is 0: **latch stores 0**

- To write a new value:
 - Set S to 0 momentarily (R stays at 1): to write a 1
 - Set R to 0 momentarily (S stays at 1): to write a 0

Given the circuit below, can outputs **S** and **R** ever simultaneously both be 0?

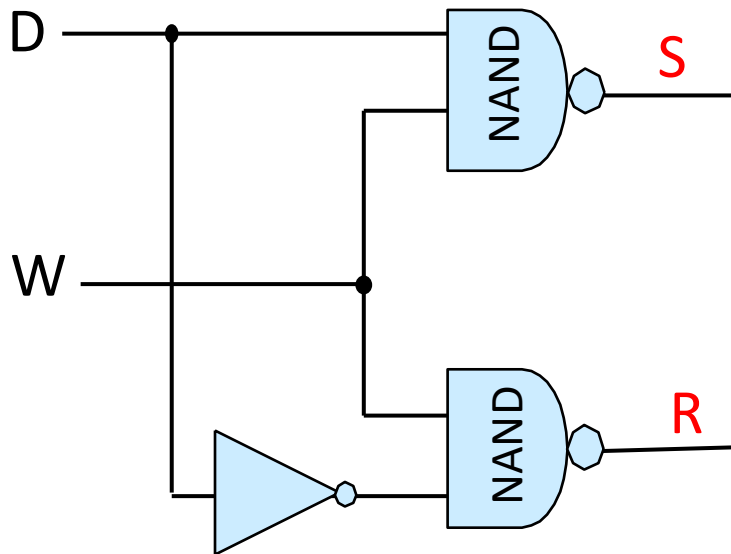


A: yes

B: no

C: maybe

Draw the truth table for this circuit



D	W	S			R		
		D	NAND	W	$\sim D$	NAND	W

When are R and S 1?

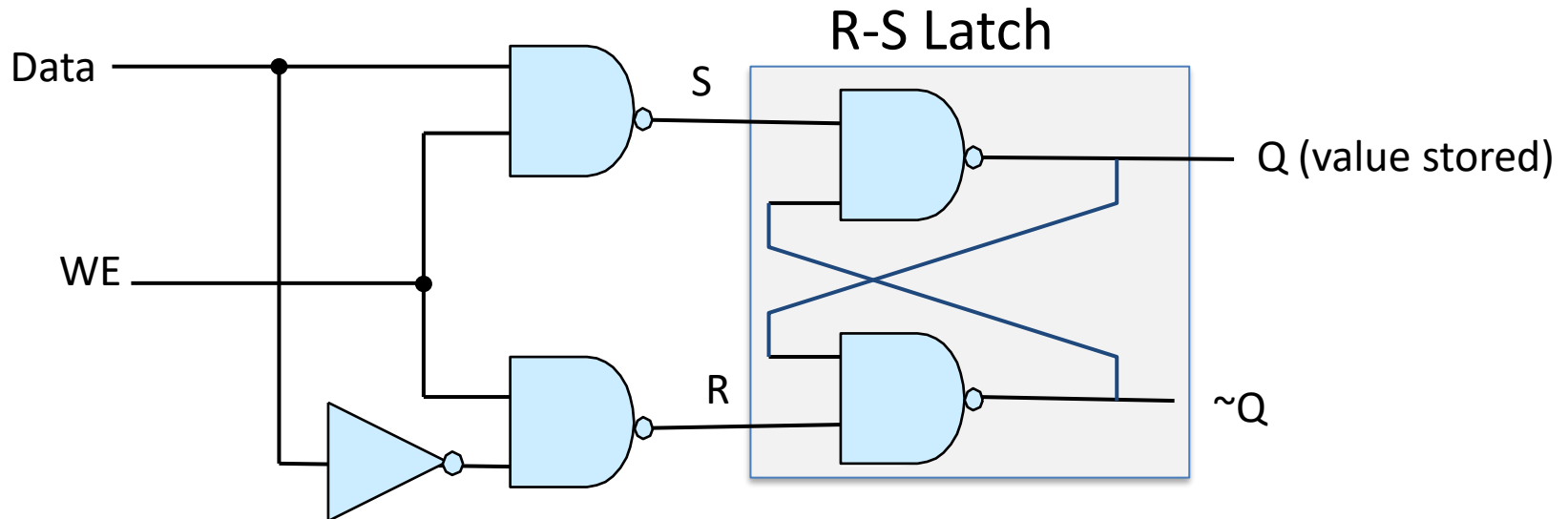
When are either R or S 0?

When is R 0?

When is S 0?

Gated D Latch

Controls R-S latch writing, ensures S & R never both 0

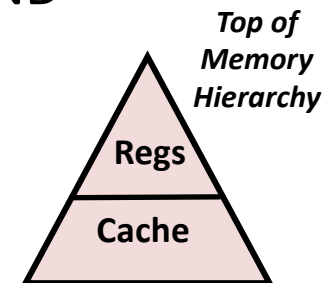


Data: data value into top NAND, $\sim$ Data into bottom NAND

WE: write-enabled, when set, latch is set to value of D

Latches are used for register and SRAM cache memory

Fast, not very dense, expensive

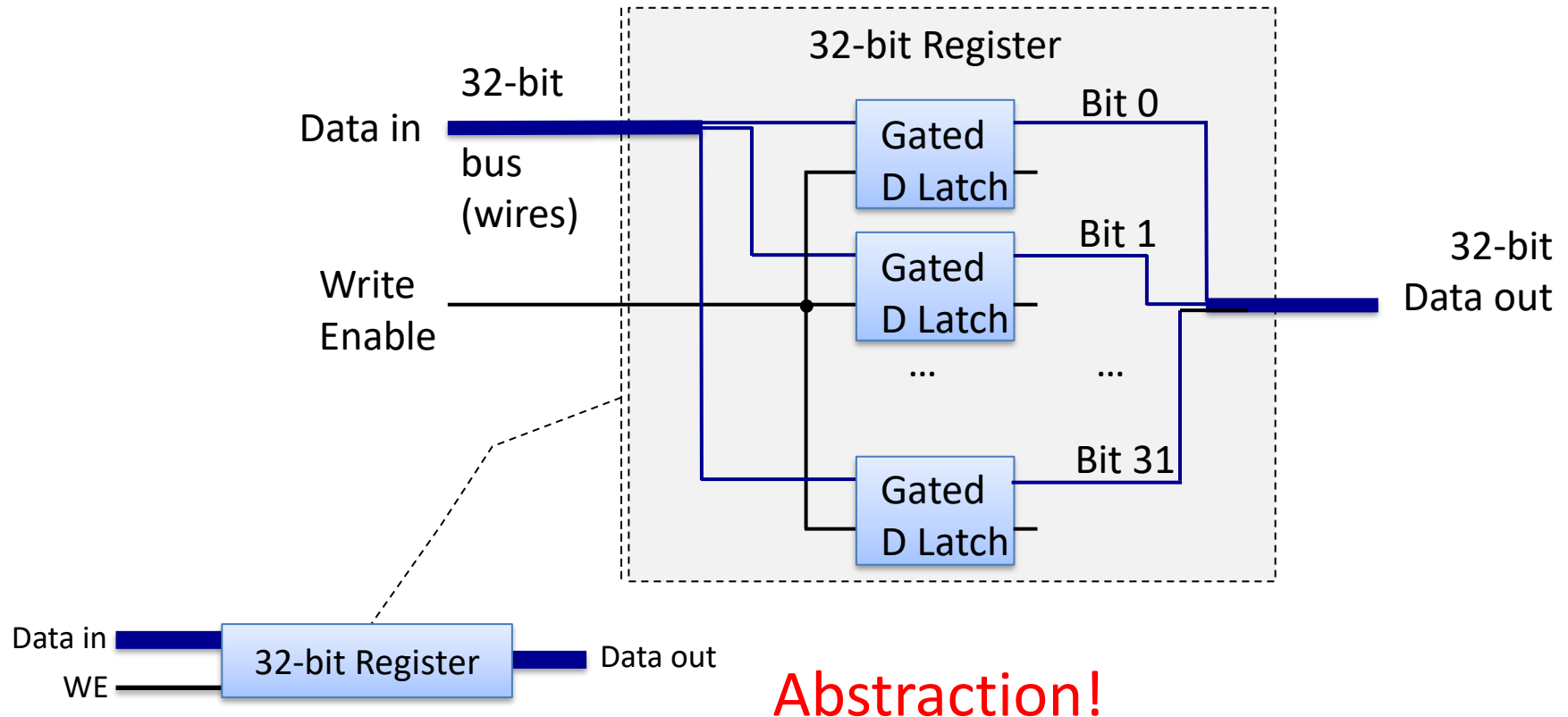


DRAM, used for RAM, is capacitor-based storage



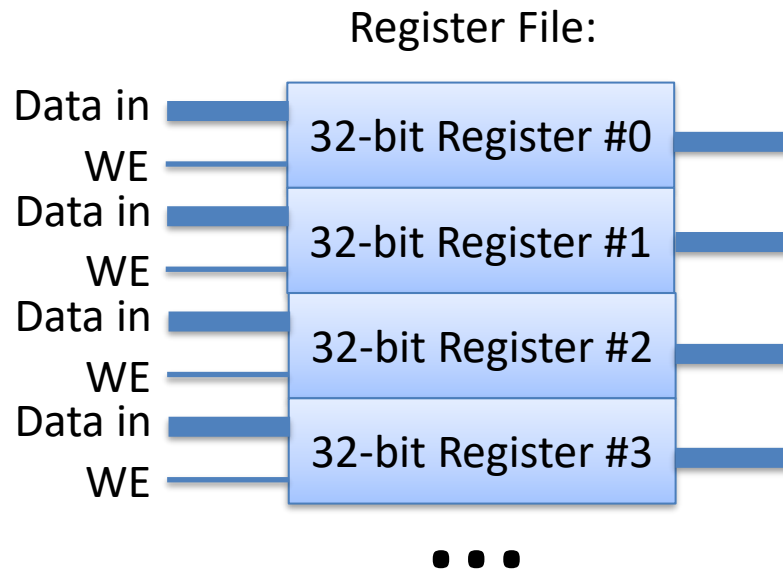
An N-bit Register

- Fixed-size storage (8-bit, 32-bit, etc.)
- Gated D latch stores one bit
 - Connect N of them to the same write-enable wire!



Register “File”

- A set of general-purpose registers for the CPU to store temporary values.



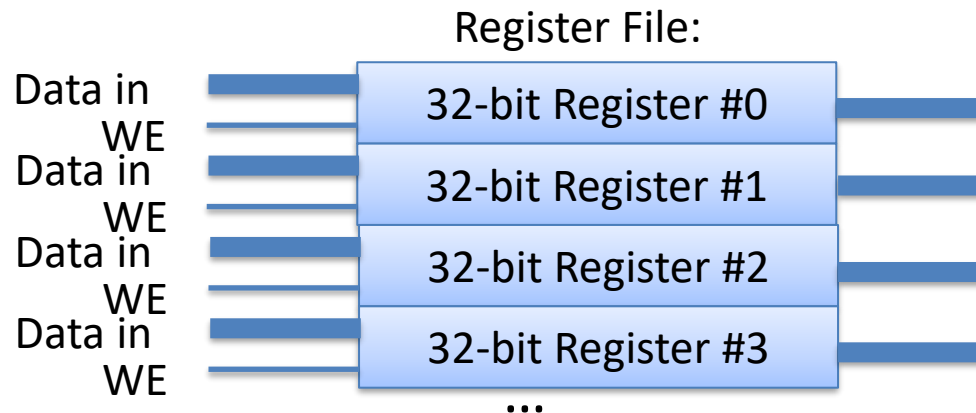
- Instructions of form:
 - “add value in **R1** and **R2**, store result in **R3**”

Register File Interface

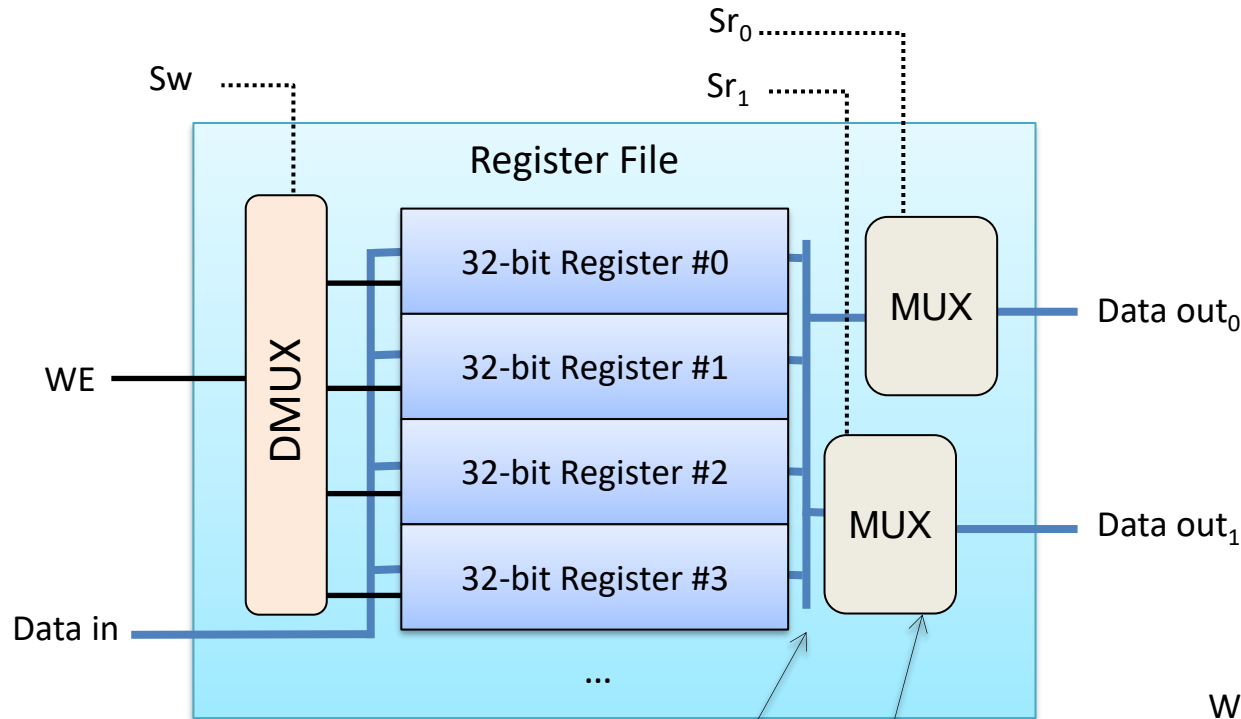
For temporary value storage of operands and results:

ADD **Rx Ry Rz** “add value in **Rx** and **Ry**,
store result in **Rz**”

- 2 Outputs: for 2 operands of ALU operation
Read Register X's value
Read Register Y's value
- 1 Data input (+ WE): for writing result to Register Z
- Need to pick two Registers for outputs (**Rx** and **Ry**)
- Need to enable exactly one WE to Register (**Rz**)



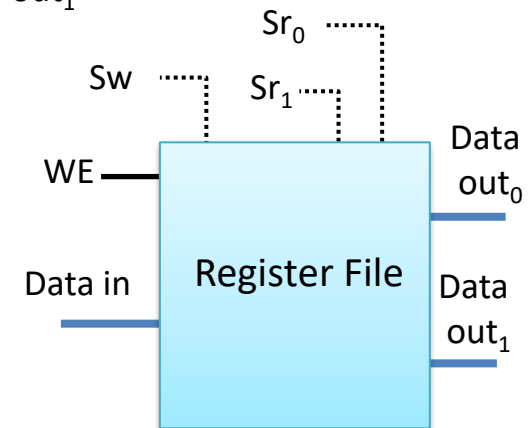
- Need to pick two Registers for outputs (**R_x** and **R_y**)
- Need to enable exactly one WE to Register (**R_z**)



All registers values are input to both MUXes

Each MUX chooses one register's value for output (input to ALU)

DMUX:
inverse of Multiplexer
1 input, N outputs
+ select input chooses one of N outputs to send input



Summary: Storage Circuits

Lots of abstraction going on here!

- Logic Gates hide the details of transistors (the building blocks of Logic Gates)
- Build 1 bit gated D latches from basic logic gates.
- Combine multiple latches to get one N-bit register.
- Grouping N-bit registers gives us register file.

Register File
N bit (32-bit) Register
1 bit gated D Latch
Basic Logic Gates
Transistors

Summary CPU so far...

We know how to store data (in register file).

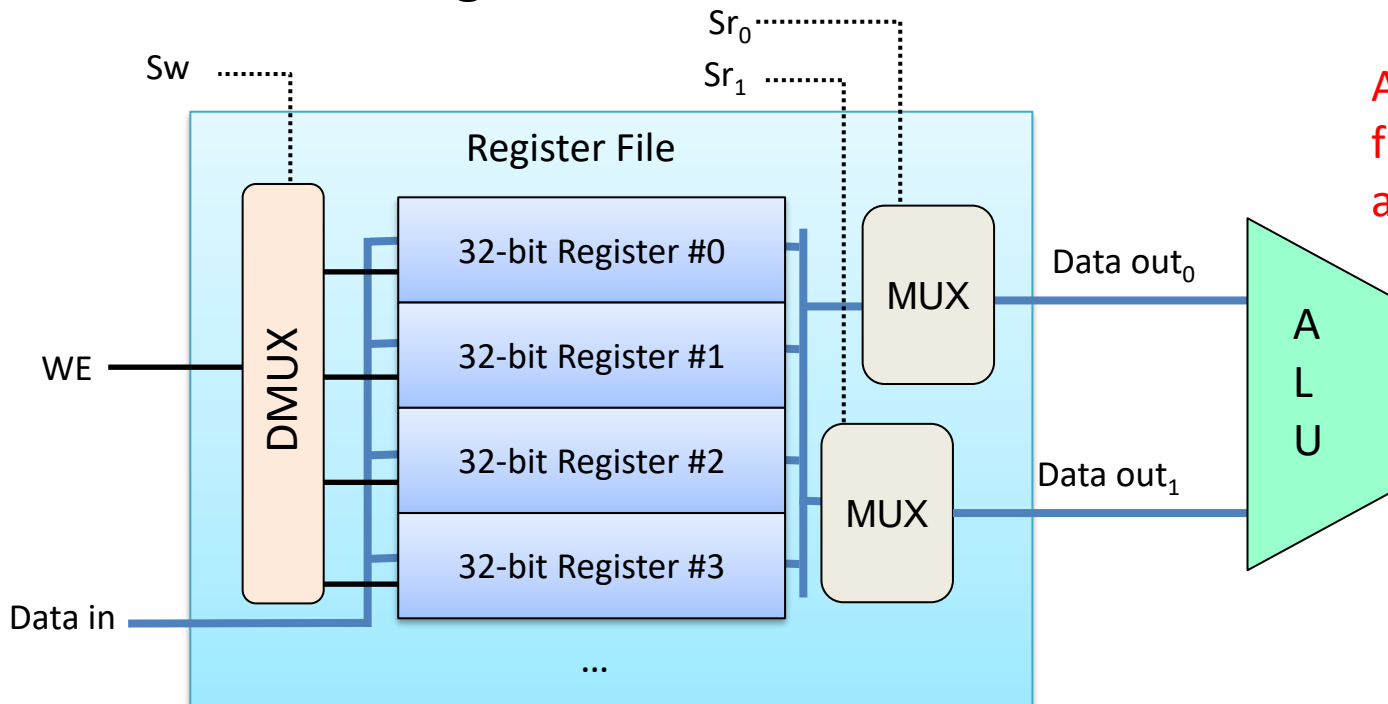
We know how to perform arithmetic on it, by feeding it to ALU.

Remaining questions:

Which register(s) do we use as input to ALU?

Which operation should the ALU perform?

To which register should we store the result?



All this info comes from the program: a series of instructions.

Summary: Building a CPU

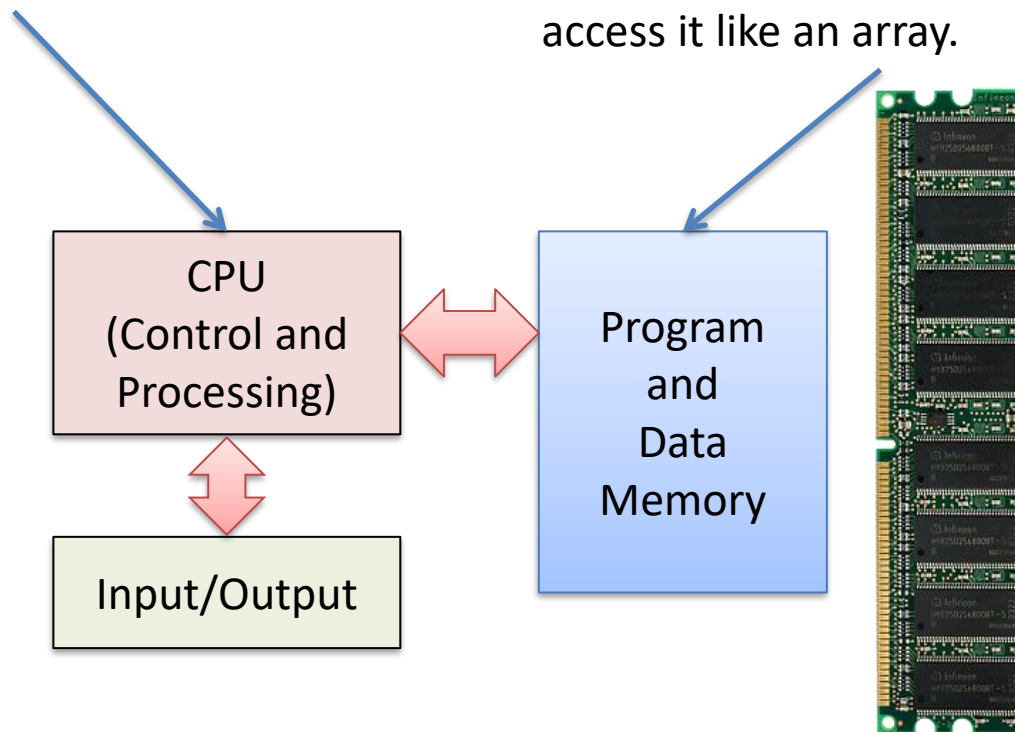
Three main classifications of HW circuits:

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(ex) Register File: set of CPU registers
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(ex) fetch the next instruction to execute

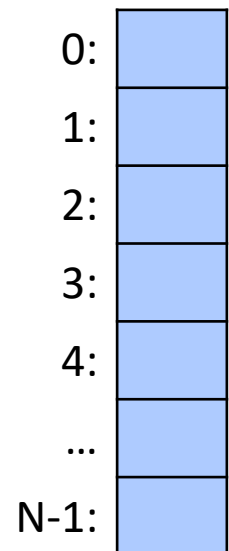
Building a CPU: Control

We're building this.

Program (instructions) live here.
We'll assume for now that we can
access it like an array.



Mem Addresses



Recall the Von Neuman model

Recall: executing instructions

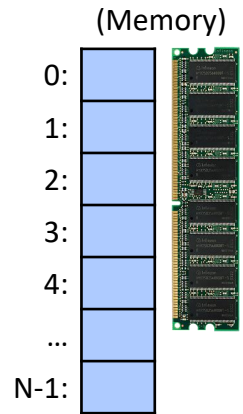
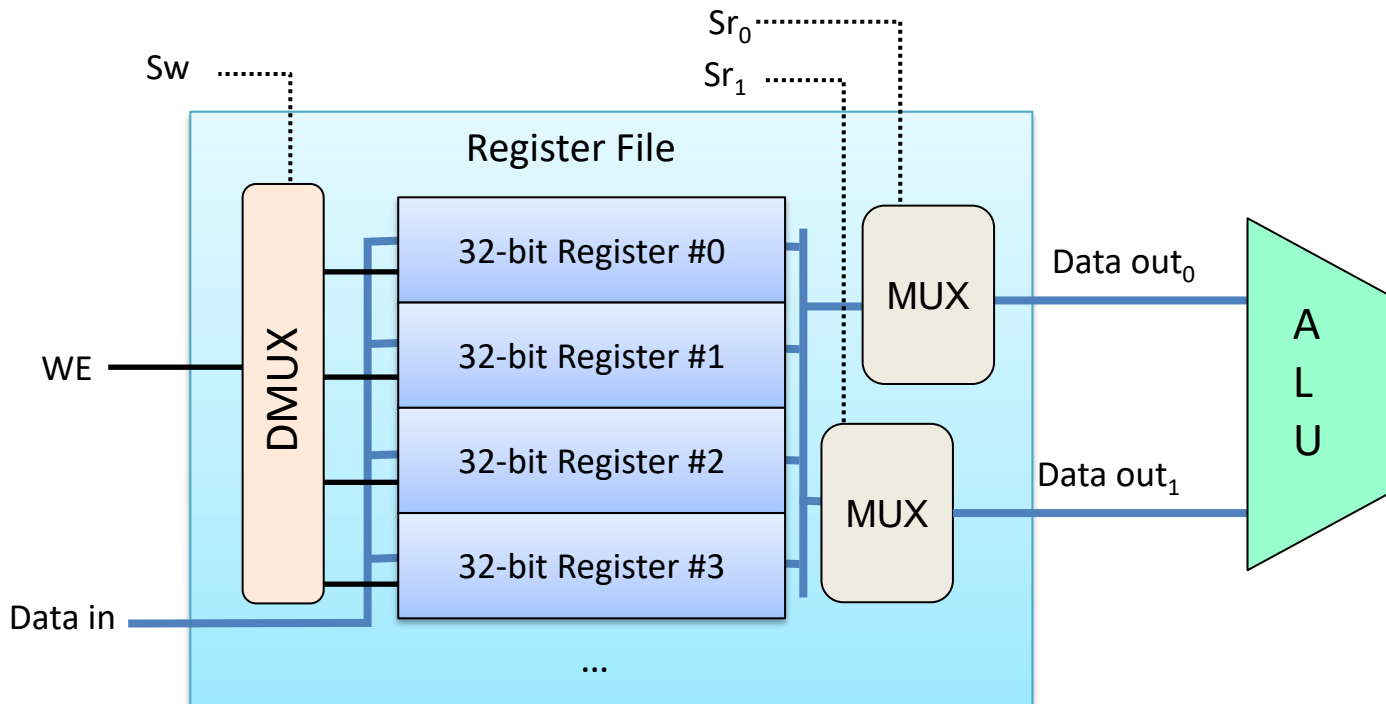
1. **Fetch** instruction from memory
2. **Decode** what the instruction is telling us to do
 - Tell the ALU what it should be doing
 - Find the correct operands
3. **Execute** the instruction (arithmetic, etc.)
4. **Store (Write Back)** the result

Program State

Let's add two more special registers (not in register file) to keep track of program.

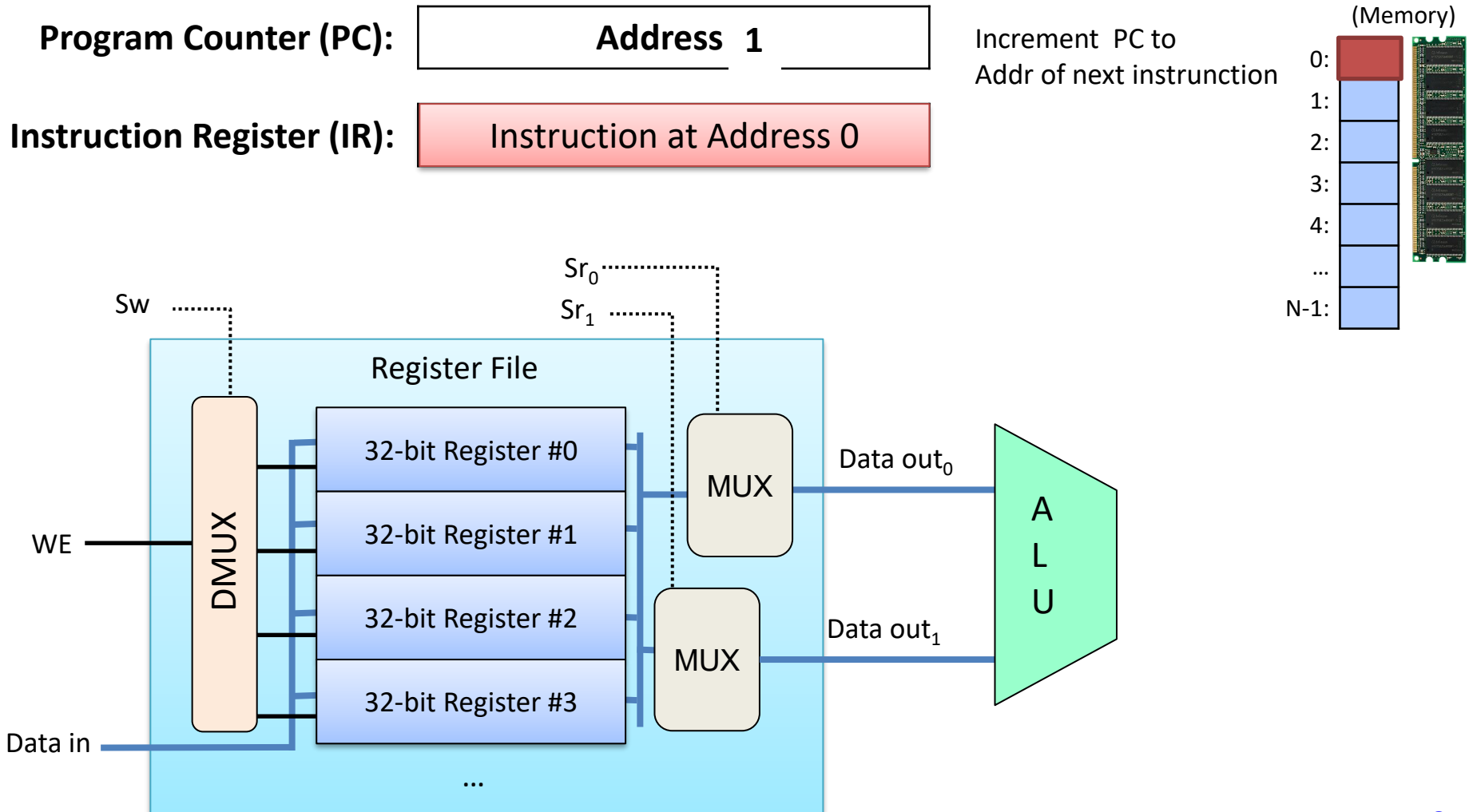
Program Counter (PC): Memory address of next instr

Instruction Register (IR): Instruction contents (bits)



Fetching instructions.

Load IR with the contents of memory at the address stored in the PC.



Decoding instructions.

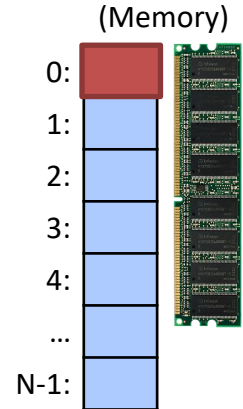
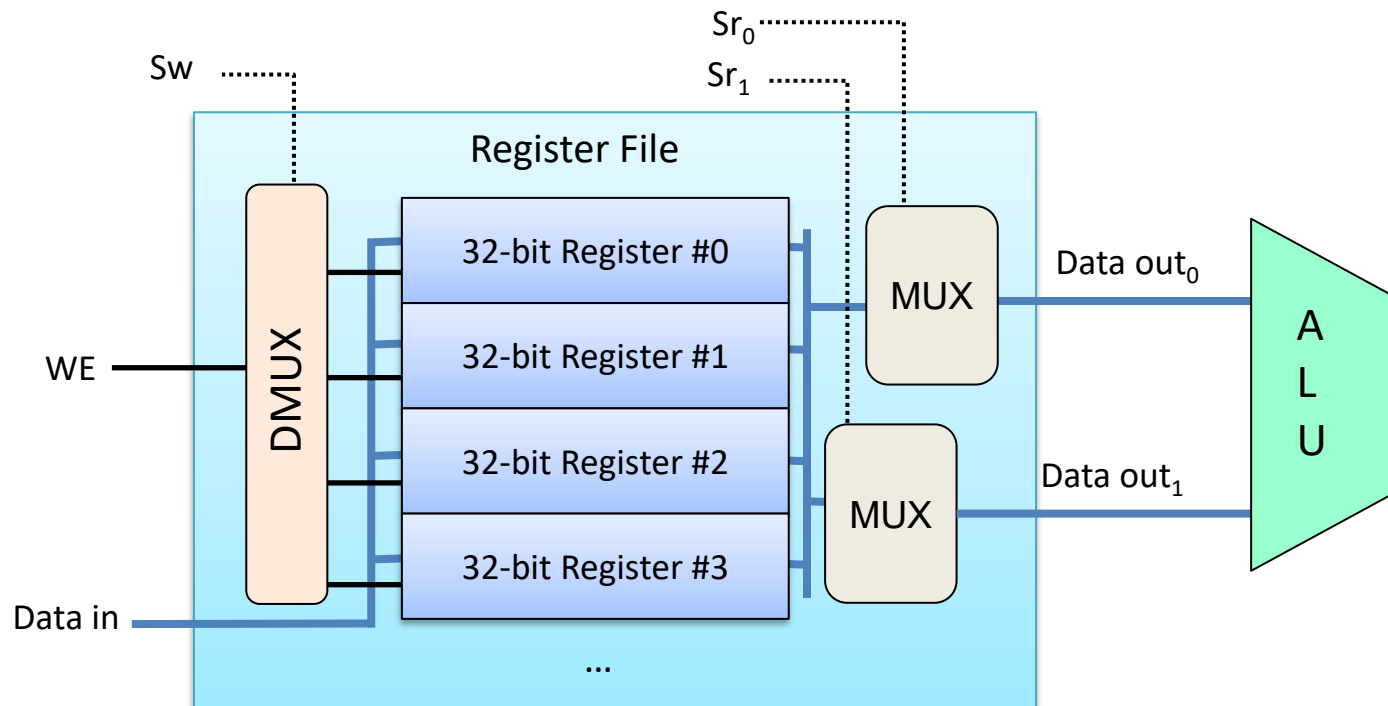
Interpret the instruction bits: What operation? Which arguments?

Program Counter (PC):

Address 1

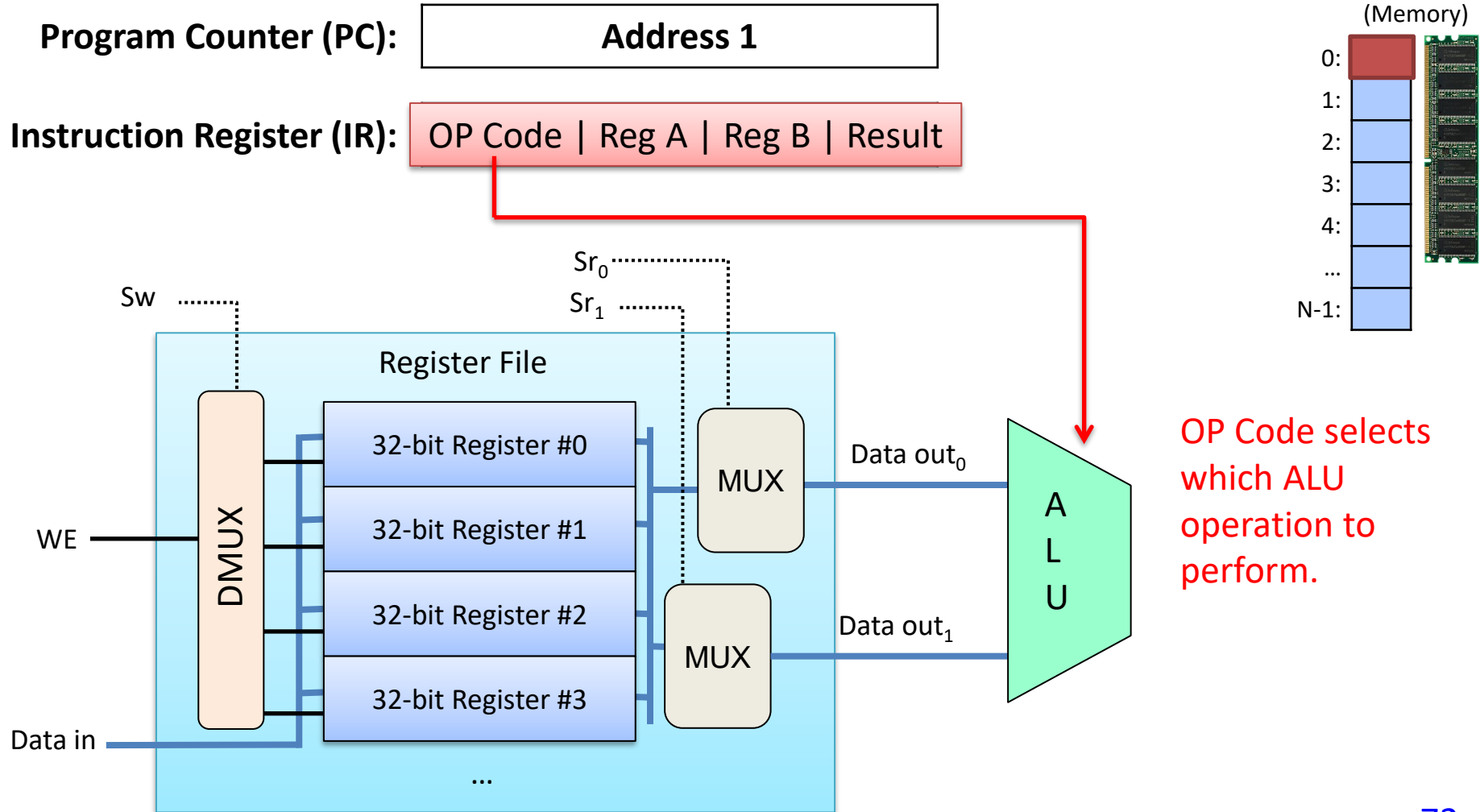
Instruction Register (IR):

OP Code | Reg A | Reg B | Result



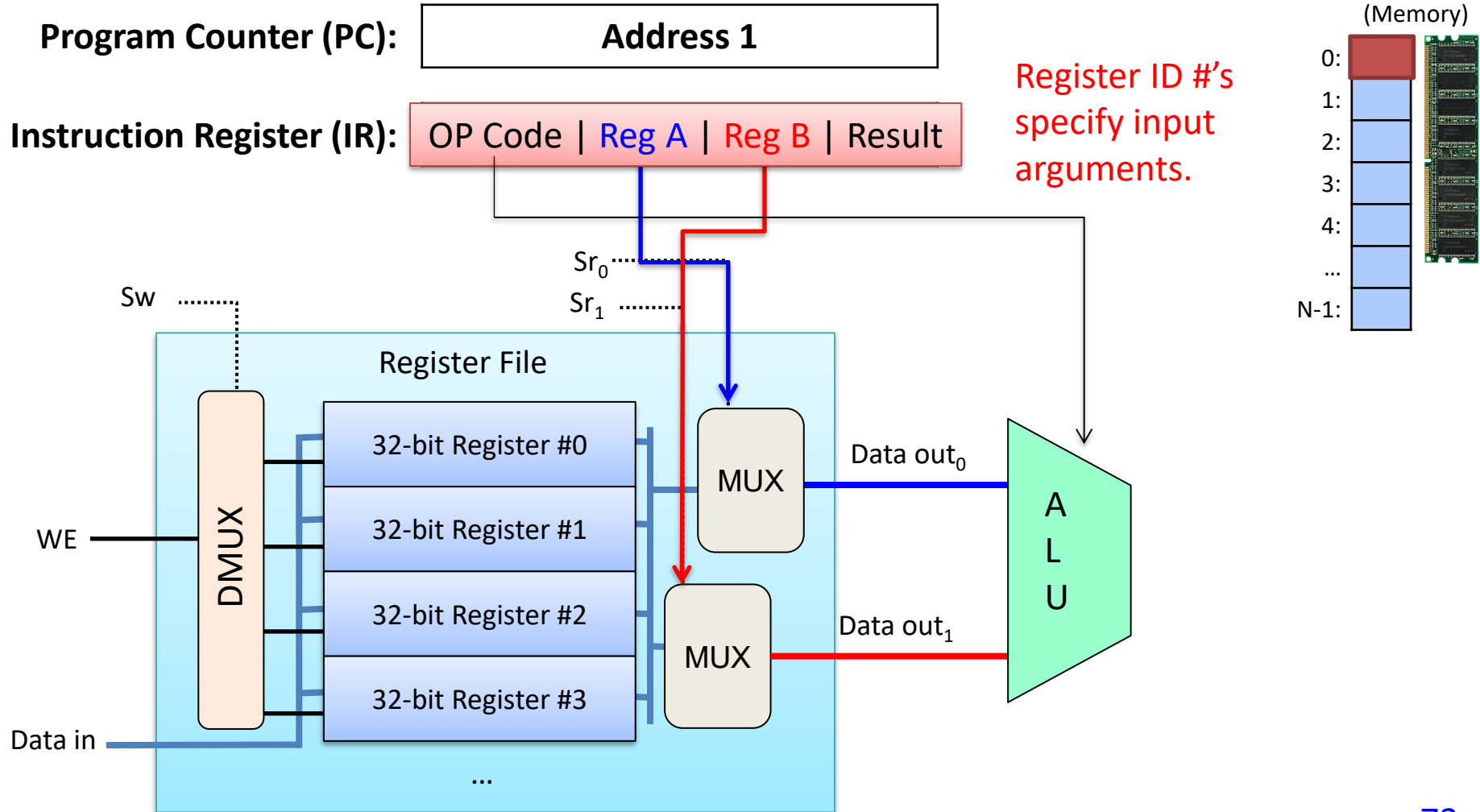
Decoding instructions.

Interpret the instruction bits: What operation? Which arguments?



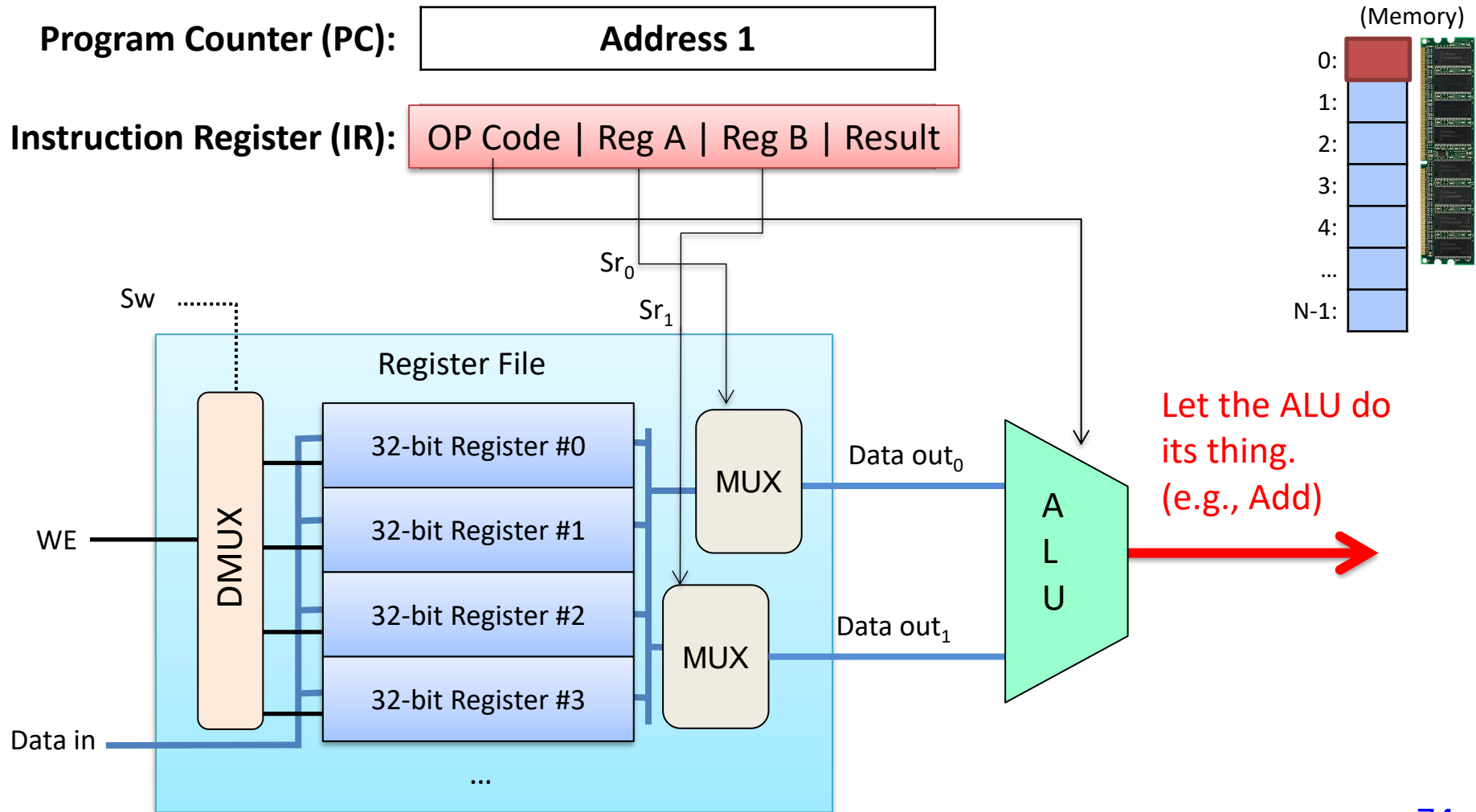
Decoding instructions.

Interpret the instruction bits: What operation? Which arguments?



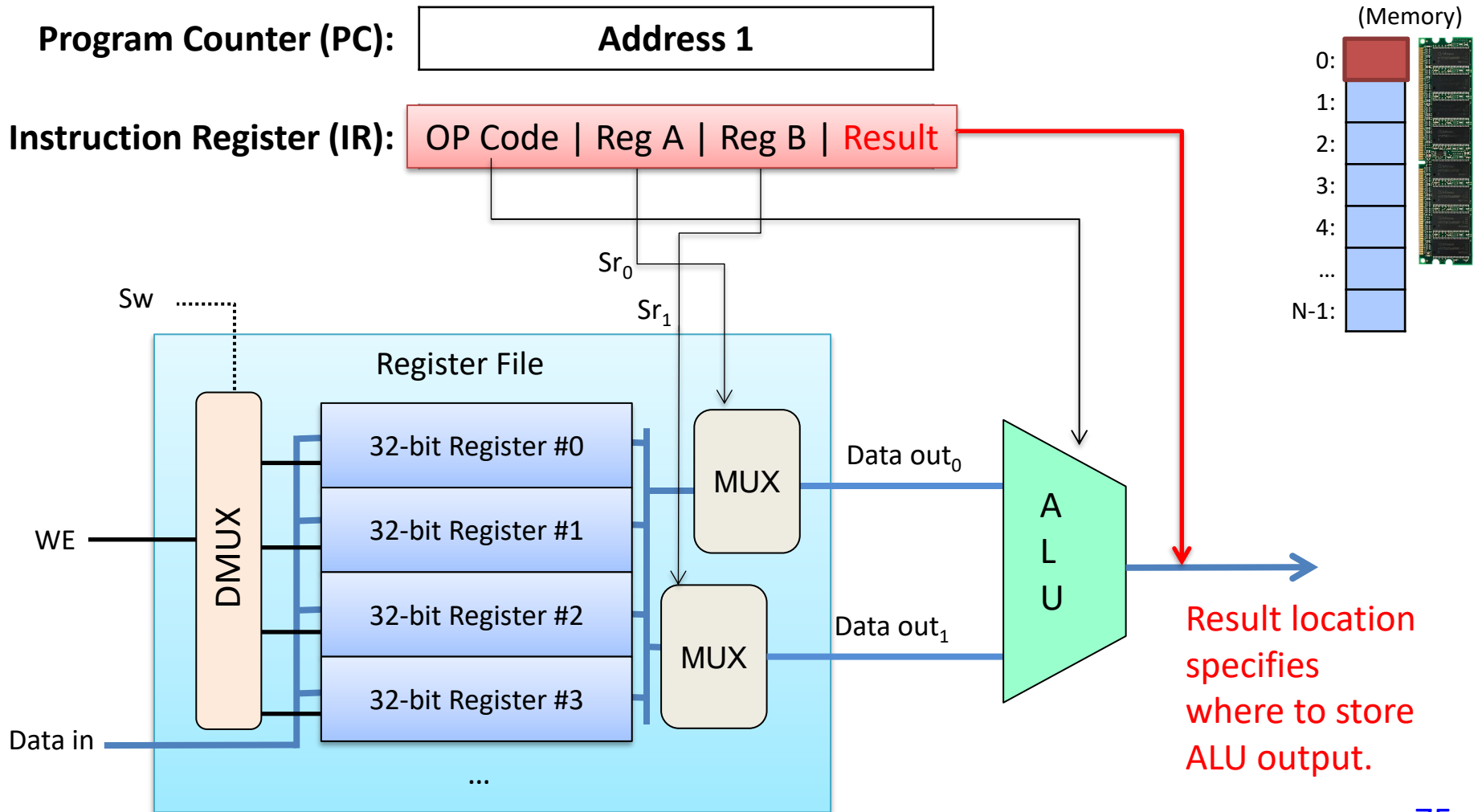
Executing instruction.

Let the ALU perform the operation on the selected operands



Storing Result

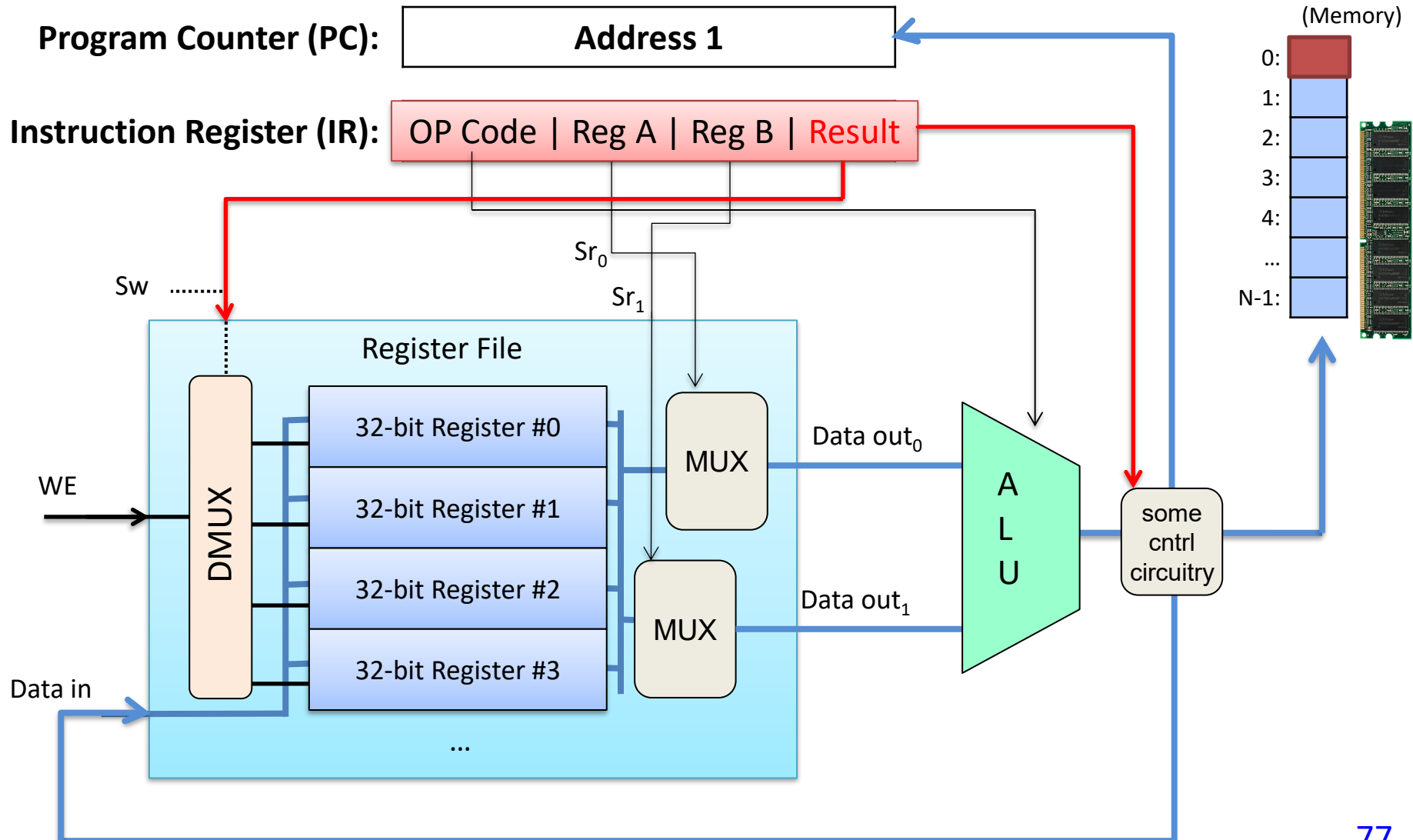
ALU has just computed a result. Where to put ALU output?



Discuss: Why do we need a program counter? Can't we just start at 0 and count up one at a time from there?

Storing Result

Interpret the instruction bits: Store result in register, memory, PC.

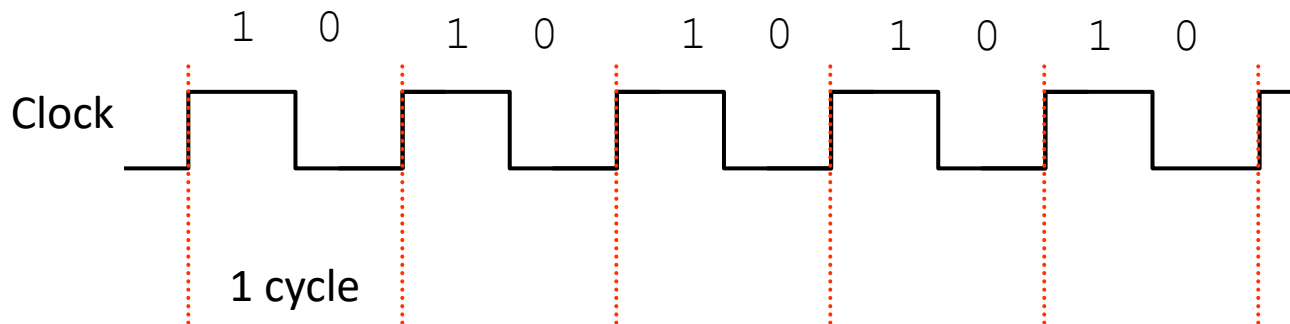


Clocking

- Need to periodically transition from one instruction to the next.
- It takes time to fetch from memory, for signal to propagate through wires, etc.
 - Too fast: don't fully compute result
 - Ripple carry adder: needs to get all the way to the end
 - Too slow: waste time

Clock Driven System

- Everything in is driven by a discrete clock
 - clock: an oscillator circuit, generates hi-low pulse
 - clock cycle: one hi-low pair

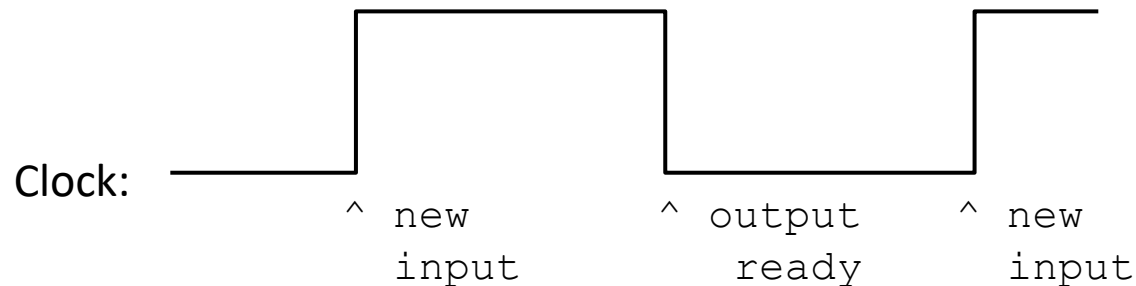


- Clock determines how fast system runs
 - Processor can only do one thing per clock cycle
 - Usually just one part of executing an instruction
 - 1GHz processor:
 - 1 billion cycles/second $\rightarrow$ 1 cycle every nanosecond (ns)
 - (1ns = 10^{-9} secs)

Clock and Circuits

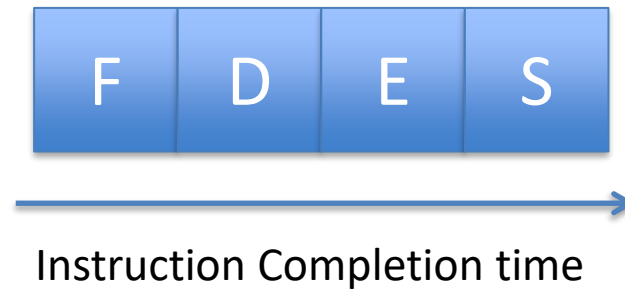
Clock Edges Triggers events

- Circuits have continuous values
- Rising Edge: trigger new input values
- Falling Edge: consistent output ready to read
- Between rising and falling edge can have inconsistent state as new input values flow through circuit



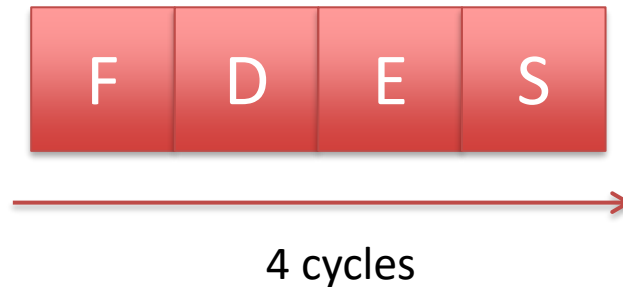
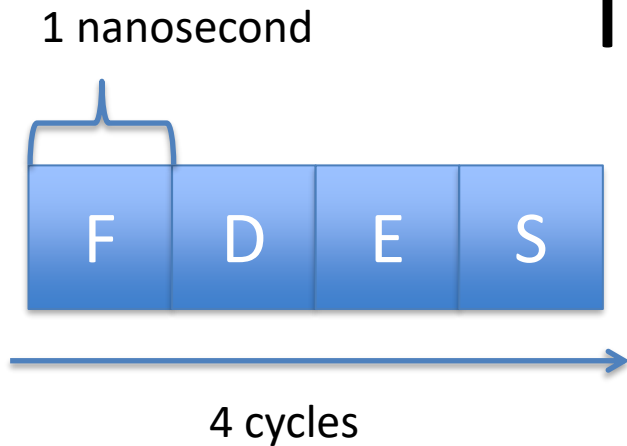
Clock cycle Time

- Execution stages: fetch, decode, execute, store

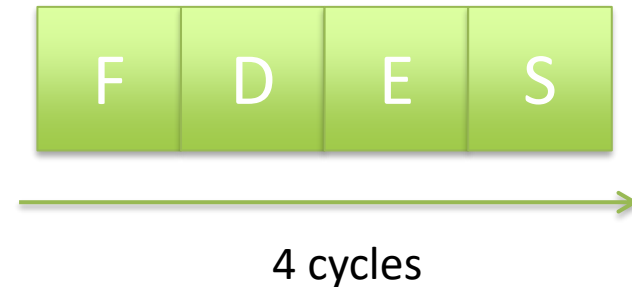


- If one stage per cycle:
longest stage to execute determines cycle time
(ex) if **E**xecute stage is longest, and takes 1 nanosec (10^{-9} sec),
then need clock cycle of 1 nanosecond (1 GHz clock)

Instruction Execution



Complete 3 instruction in 12 cycles
(or 3 instructions in 12 nanoseconds)



Q: Can we do better?

Analogy

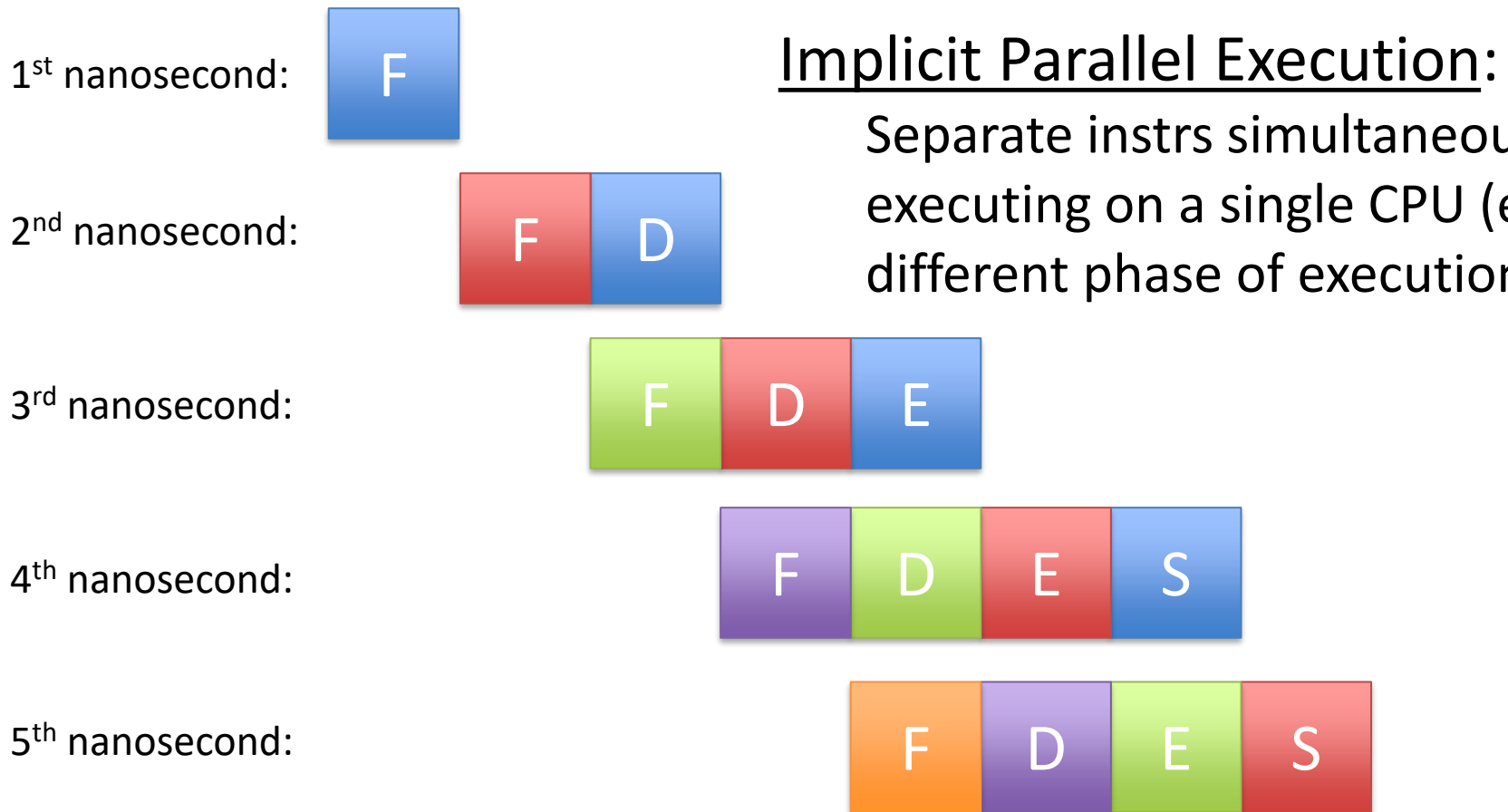
- Laundry
 - 4 Steps per load:
Wash first, then **D**ry, then **F**old, then **P**ut Away
 - If have 6 loads, what do you do when you take first load out of the washer?



Start **W**ashing the second Load while first is in **D**ryer

Pipelining (CPU)

Idea: feed next instruction into previous stage of execution



Implicit Parallel Execution:

Separate instrs simultaneously executing on a single CPU (each in different phase of execution)

Steady state: One instruction finishes every nanosecond!

Summary: Building a CPU

Three main classifications of HW circuits:

1. ALU: implement arithmetic & logic functionality
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2. Storage: to store binary values
(ex) Register File: set of CPU registers
3. Control: support/coordinate instruction execution
(ex) fetch the next instruction to execute

Summary: Architecture & Circuits

Modern Computer based on Von Neumann

- Generic Compute Machine & Stored Program model

To run program, need different types of circuits:

- ALU: addition, subtraction, multiplexes, and, or, etc
- storage: R-S latches, registers
- control: (ex) fetch next instr from RAM
- abstractions allow us to build more complex functionality from simpler functionality

Clock-driven system (discrete time)

Pipelining: overlap instruction execution